STANDARDS

The calculations are carried out in accordance with:

- Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings.
- Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings.
- Eurocode 3: Design of steel structures. Part 1-8: Design of joints.
- EN 10080: Steel for the reinforcement of concrete. Weldable reinforcing steel. General.

For all NDPs (Nationally Determined Parameter) in the Eurocodes the recommended values are used.

NDP's are as follows:

Parameter	γ_c	γ_s	α_{cc}	α_{ct}	$C_{Rd,c}$	v_{min}	k_1
Recommended value	1.5	1.15	1.0	1.0	0.12	$0.035k^{1/3}\cdot f_{ck}^{1/2}$	0.15

Table 1: NDP-s in EC-2.

Parameter	γ_{M0}	γ_{M1}	γ_{M2}
Recommended value	1.0	1.0	1.25

Table 2: NDP-s in EC-3.

QUALITIES

Concrete grade C35/45:

$f_{ck} = 35{,}0\ \text{MPa}$	EC2, Table 3.1
$f_{cd} = \alpha_{cc}\cdot f_{ck}/\gamma_c = 1\cdot 35/1{,}5 = 23{,}3\ \text{MPa}$	EC2, Pt.3.15
$f_{ctd} = \alpha_{ct}\cdot f_{ctk,0,05}/\gamma_c = 1\cdot 2{,}20/1{,}5 = 1{,}46\ \text{MPa}$	EC2, Pt.3.16
$f_{bd} = 2{,}25\cdot\eta_1\cdot\eta_2\cdot f_{ctd} = 2{,}25\cdot 0{,}7\cdot 1{,}0\cdot 1{,}46 = 2{,}3\ \text{MPa}$	EC2, Pt.8.4.2

$f_{ck} := 35\cdot\text{MPa} = 5076.321\cdot\text{psi}$	Characteristic Cylinder Strength
$f_{cd} := 1.0\cdot\frac{f_{ck}}{1.5} = 3384.214\cdot\text{psi}$	Design Compressive Strength
$f_{ctk0\_05} := 2.2\cdot\text{MPa}$	Characteristic axial tensile strength of concrete
$f_{ctd} := 1.0\cdot\frac{f_{ctk0\_05}}{1.5} = 212.722\cdot\text{psi}$	Design Tensile Strength

η_1 is a coefficient related to the quality of the bond condition and the position of the bar during concreting
= 1.0 for condition of good bond
= 0.7 for all other cases and for bars in structural elements built with slipforms

η_2 is related to bar diameter
= 1.0 for $\phi \le 40\text{mm}$ (NDP)
= $(140 - \phi)/100$ for $\phi > 40\text{mm}$

$\eta_1 := 0.7$	coefficient related to bond condition
$\eta_2 := 1.0$	coefficient related to bar diameter
$d_{rb} := 0.5\cdot\text{in} = 12.7\cdot\text{mm}$	Use #4 Rebar
$f_{bd} := 2.25\cdot\eta_1\cdot\eta_2\cdot f_{ctd} = 335.037\cdot\text{psi}$	Design Bond Stress

Reinforcement B500C:

$f_{yd} = f_{yk}/\gamma_s = 500/1,15 = 435$ MPa EC2, Pt.3.2.7

$$f_{yd} := 500 \cdot \frac{\mathrm{MPa}}{1.15} = 63059.886 \cdot \mathrm{psi}$$

Design Yield Strength of Reinforcement

Structural steel S355:

Tension: $f_{yd} = f_y/\ \gamma_{M0} = 355/1,0 = 355$ MPa
Compression: $f_{yd} = f_y/\ \gamma_{M0} = 355/1,0 = 355$ MPa
Shear: $f_{sd} = f_y/(\gamma_{M0} \cdot \sqrt{3}) = 355/(1,0 \cdot \sqrt{3}) = 205$ MPa

$$f_{yd\_ts} := 355 \cdot \frac{\mathrm{MPa}}{1.0} = 51488.397 \cdot \mathrm{psi}$$

Design Tension Stress of Tube Steel

$$f_{yd\_ts} := 355 \cdot \frac{\mathrm{MPa}}{1.0} = 51488.397 \cdot \mathrm{psi}$$

Design Compression Stress of Tube Steel

$$f_{sd\_ts} := 355 \cdot \frac{\mathrm{MPa}}{1.0 \cdot \sqrt{3}} = 29726.84 \cdot \mathrm{psi}$$

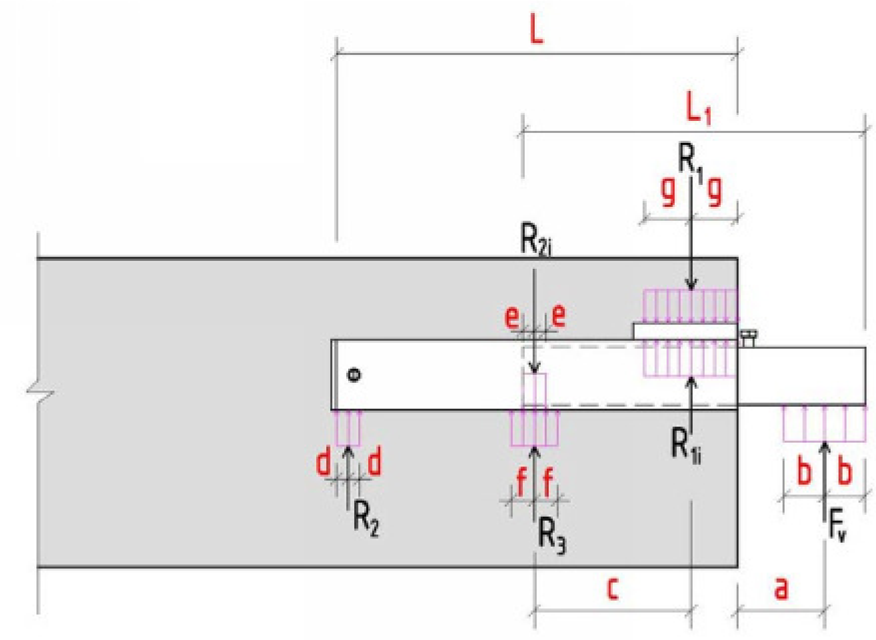
Design Shear Stress of Tube Steel

DIMENSIONS

Inner tube: HUP 100x50x6, Cold formed, S355
Outer tube: HUP 120x60x4, Cold formed, S355

LOADS

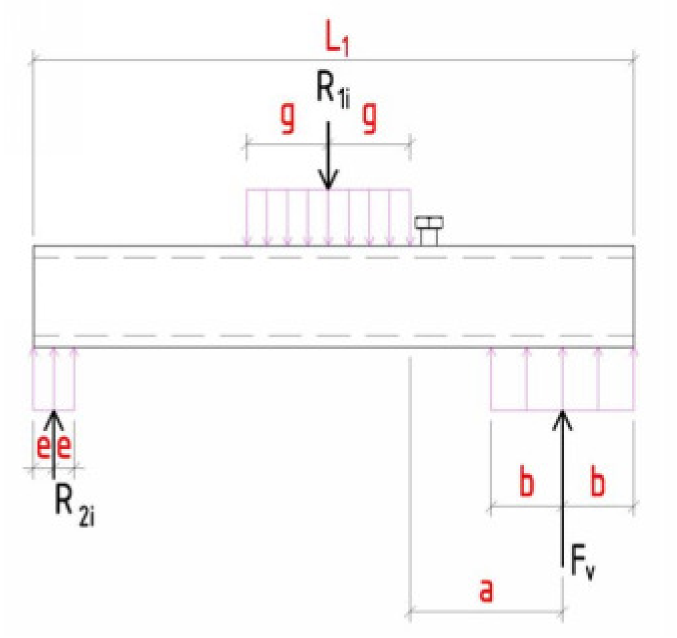
Vertical ultimate limit state load = F_V = 100kN. $F_V := 100 \cdot \mathrm{kN} = 22.481 \cdot \mathrm{kip}$



F_V = External force on the inner tube

R_{1i}, R_{2i} = Internal forces between the inner and outer tubes.
R_1, R_2, R_3 = Support reaction forces the outer tube.

g= distance to the middle plane of the anchoring stirrups in front of the unit.



Equilibrium equations of the inner tube:

1): $\sum M=0$: $F_V \cdot (L_1 - b - e) - R_{1i} \cdot (L_1 - b - a - g - e) = 0$
2): $\sum F_y=0$: $F_V - R_{1i} + R_{2i} = 0$

Assuming Nominal Values:

$L_1 := 295 \cdot mm$	$a := 75 \cdot mm$	$b := 35 \cdot mm$	$g := 40 \cdot mm$	$e := 10 \cdot mm$
$L_1 = 11.614\,in$	$a = 2.953\,in$	$b = 1.378\,in$	$g = 1.575\,in$	$e = 0.394\,in$

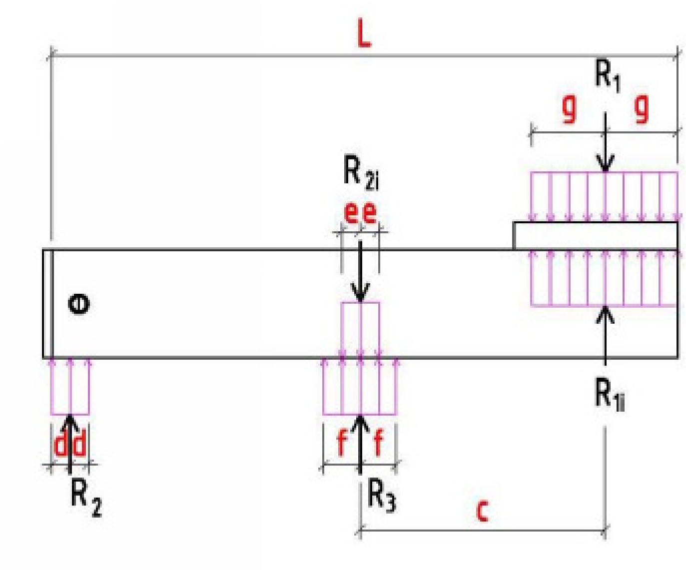
Results:

$$R_{1i} = \frac{100kN \cdot (295 - 35 - 10)mm}{(295 - 35 - 75 - 40 - 10)mm} = 185.2kN$$

$$R_{2i} = 185.2kN - 100kN = 85.2kN$$

$R_{1i} := \dfrac{F_V \cdot (L_1 - b - e)}{(L_1 - b - a - g - e)}$ $R_{1i} = 185.185 \cdot kN$ $R_{1i} = 41.631 \cdot kip$

$R_{2i} := R_{1i} - F_V$ $R_{2i} = 85.185 \cdot kN$ $R_{2i} = 19.15 \cdot kip$



Exact distribution of forces depends highly on the behavior of the outer tube. Both longitudinal bending stiffness and local transverse bending stiffness in the contact points between the inner and the outer tubes affects the equilibrium. Two situations are considered:

1) Rigid outer tube.

Outer tube rotates as a stiff body. This assumption gives minimum reaction force at R_1, and maximum reaction force at R_2. R_3 becomes zero. (The force R_3 will actually be negative, but since no reinforcement to take the negative forces is included at this position, it is assumed to be zero.)

Equilibrium equations of the outer tube:

1): $\sum M=0$: $(R_{1i}-R_1)\cdot(L-3-g-d)-(R_{2i}-R_3)\cdot(L-3-g-c-d)=0$ (5)

2): $\sum F_y=0$: $R_2+R_3+R_{1i}-R_{2i}-R_1=0$ (6)

Assuming Nominal Values:

$L := 348\cdot mm$ $c := L_1 - b - a - g - e = 135\cdot mm$ $g := 40\cdot mm$ $e := 10\cdot mm$ $d := 10\cdot mm$

$L = 13.701 in$ $c = 5.315 in$ $g = 1.575 in$ $e = 0.394 in$ $d = 0.394 in$

Assume $R3_1 := 0\cdot kip$ per discussion above

$$(185.2 - R_1)\cdot(348-3-40-10)-(85.2-0)\cdot(348-3-40-135-10)=0$$

$$54634 - 295R_1 - 13632 = 0$$

$$R_1 = \frac{41002}{295} = 139kN$$

Given

$$(R_{1i} - R_1)\cdot(L - 3\cdot mm - g - d) - (R_{2i} - R_3)\cdot(L - 3\cdot mm - g - c - d) = 0$$

$R1_1 := Find(R_1)$ $R1_1 = 138.983\cdot kN$ $R1_1 = 31.245\cdot kip$

$$R_2 = R_1 + R_{2i} - R_{1i} - R_3 = 139 + 85.2 - 185.2 = 39kN$$

$R2_1 := R1_1 + R_{2i} - R_{1i} - R3_1$ $R2_1 = 38.983\cdot kN$ $R2_1 = 8.764\cdot kip$

2) Outer tube without bending stiffness. No forces transferred to outer tube at the back of inner tube.

This assumption gives maximum reaction forces R_1 and R_3. R_2 becomes zero. The forces follow directly from the assumption: $R_1 = R_{1i}$ $R_3 = R_{2i}$ and $R_2 = 0$

$$R_1 = 185.2kN$$
$$R_2 = 0kN$$
$$R_3 = 85.2kN$$

$R1_2 := R1i$ $\quad R1_2 = 185.185 \cdot kN$ $\quad R1_2 = 41.631 \cdot kip$

$R2_2 := 0 \cdot kip$

$R3_2 := R2i$ $\quad R3_2 = 85.185 \cdot kN$ $\quad R3_2 = 19.15 \cdot kip$

The magnitude of the forces will be somewhere in between the two limits, and the prescribed reinforcement ensures integrity for both situations. Reinforcement is to be located at the assumed attack point for support reactions.

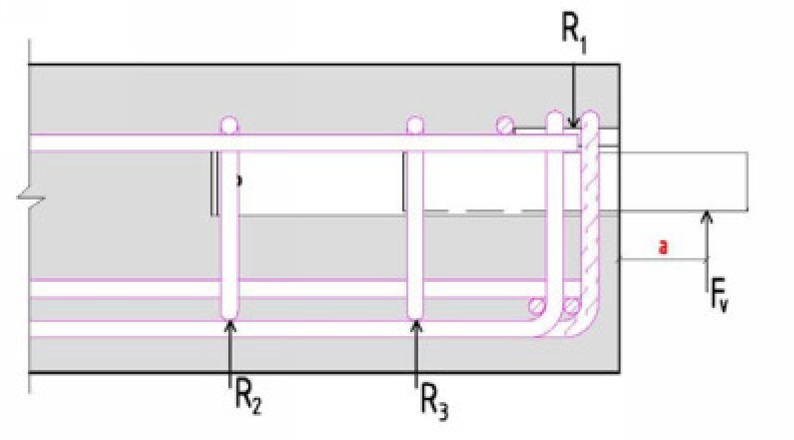
Use Maximum Reactions considering both assumptions

$R_1 := \max(R1)$ $\quad R_1 = 185.185 \cdot kN$ $\quad R_1 = 41.631 \cdot kip$

$R_2 := \max(R2)$ $\quad R_2 = 38.983 \cdot kN$ $\quad R_2 = 8.764 \cdot kip$

$R_3 := \max(R3)$ $\quad R_3 = 85.185 \cdot kN$ $\quad R_3 = 19.15 \cdot kip$

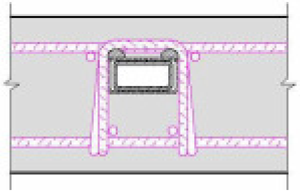
Reinforcement Necessary to anchor the unit to concrete



Eurocode Equations

Reinforcement for $R_1 = 185.185 \cdot kN$

Reinforcement R_1: $A_{s1} = R_1/f_{sd} = 185.2kN/435Mpa = 426\ mm^2$
Select 2-Ø12 = 2×2×113 =452 mm^2
Capacity selected reinforcement: R=452 mm^2 ·435MPa=196.6kN



$A_{s1} := \frac{R_1}{f_{yd}}$ $\quad A_{s1} = 425.926 \cdot mm^2$

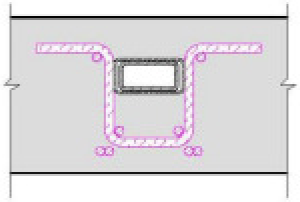
Bar Size $\quad size_{metric} := 12$

Bars Required $\quad N_{reqd} := \mathrm{ceil}\left(\frac{A_{s1}}{A_{rb\_metric}{}_{size_{metric}} \cdot 2}\right)$ $\quad N_{reqd} = 2$

Capacity of Supplied Reinforcing $\quad N_{reqd} \cdot \left(A_{rb\_metric}{}_{size_{metric}} \cdot 2 \cdot f_{yd}\right) = 196.522 \cdot kN$

Reinforcement for $R_3 = 85.185 \cdot kN$

Reinforcement R$_3$: $A_{s3} = R_3/f_{sd} = 85.2kN/435MPa = 196\ mm^2$
Select 1-Ø12 = $1 \times 2 \times 113 = 226\ mm^2$
Capacity selected reinforcement: R=$226\ mm^2 \cdot 435MPa = 98.3kN$



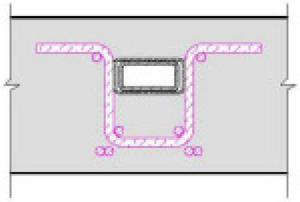
$A_{s3} := \frac{R_3}{f_{yd}}$ $A_{s3} = 195.926 \cdot mm^2$ Bar Size $size_{metric} := 12$

Bars Required $N_{reqd} := ceil\left(\frac{A_{s3}}{Arb\_metric_{size_{metric}} \cdot 2}\right)$ $N_{reqd} = 1$

Capacity of Supplied Reinforcing $N_{reqd} \cdot \left(Arb\_metric_{size_{metric}} \cdot 2 \cdot f_{yd}\right) = 98.261 \cdot kN$

Reinforcement for $R_2 = 38.983 \cdot kN$

Reinforcement R$_2$: $A_{s2} = R_2/f_{sd} = 39kN/435MPa = 89\ mm^2$
Select 1-Ø12 = $1 \times 2 \times 113 = 226mm^2$
Capacity selected reinforcement: R=$226\ mm^2 \cdot 435MPa = 98.3kN$



$A_{s2} := \frac{R_2}{f_{yd}}$ $A_{s2} = 89.661 \cdot mm^2$ Bar Size $size_{metric} := 12$

Bars Required $N_{reqd} := ceil\left(\frac{A_{s2}}{Arb\_metric_{size_{metric}} \cdot 2}\right)$ $N_{reqd} = 1$

Capacity of Supplied Reinforcing $N_{reqd} \cdot \left(Arb\_metric_{size_{metric}} \cdot 2 \cdot f_{yd}\right) = 98.261 \cdot kN$

US equivalent Equations

Reinforcement for $R_1 = 41.631 \cdot kip$

Rebar Yield Strength $f_y := 60 \cdot ksi$ Strength Reduction Factor for rebar in tension $\phi_t := 0.9$

$A_{s1} := \frac{R_1}{\phi_t \cdot f_y}$ $A_{s1} = 0.771 \cdot in^2$ Bar Size $size := 4$

Bars Required $N_{reqd} := ceil\left(\frac{A_{s1}}{Arb_{size} \cdot 2}\right)$ $N_{reqd} = 2$

Capacity of Supplied Reinforcing $N_{reqd} \cdot \left[Arb_{size} \cdot 2 \cdot \left(\phi_t \cdot f_y\right)\right] = 43.2 \cdot kip$

Reinforcement for $R_3 = 19.15 \cdot kip$

Rebar Yield Strength $f_y := 60 \cdot ksi$ Strength Reduction Factor for rebar in tension $\phi_t := 0.9$

$A_{s3} := \frac{R_3}{\phi_t \cdot f_y}$ $A_{s3} = 0.355 \cdot in^2$ Bar Size $size := 4$

Bars Required $N_{reqd} := ceil\left(\frac{A_{s3}}{Arb_{size} \cdot 2}\right)$ $N_{reqd} = 1$

Capacity of Supplied Reinforcing $N_{reqd} \cdot \left[Arb_{size} \cdot 2 \cdot \left(\phi_t \cdot f_y\right)\right] = 21.6 \cdot kip$

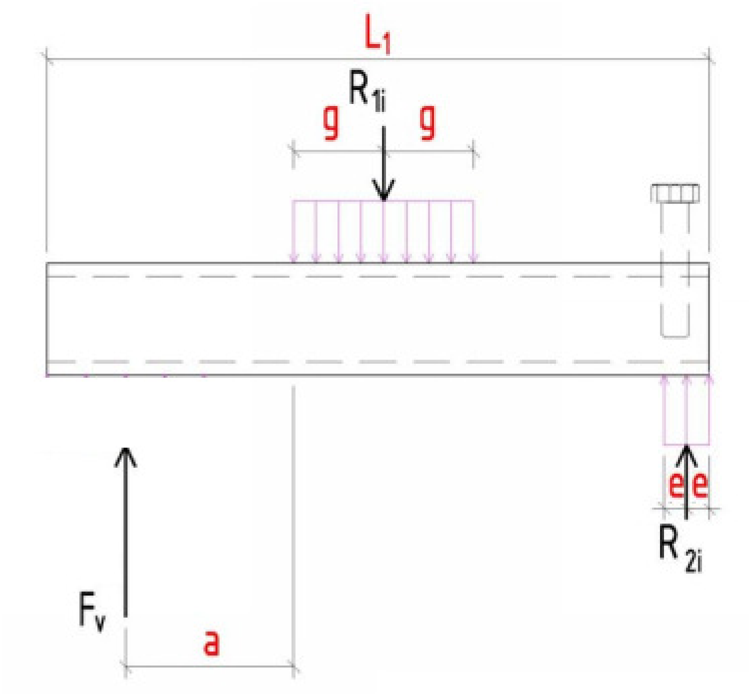
Reinforcement for $R_2 = 8.764 \cdot kip$

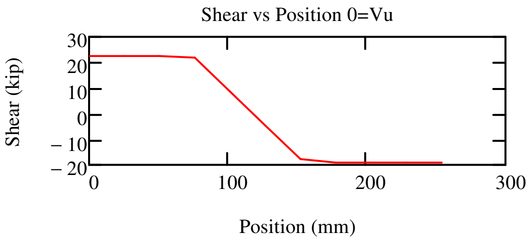
Rebar Yield Strength $f_y := 60 \cdot ksi$ Strength Reduction Factor for rebar in tension $\phi_t := 0.9$

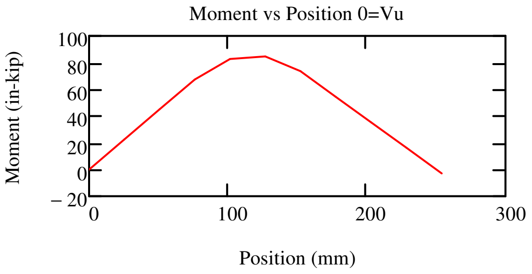
$A_{s2} := \frac{R_2}{\phi_t \cdot f_y}$ $A_{s2} = 0.162 \cdot in^2$ Bar Size $size := 4$

Bars Required $N_{reqd} := ceil\left(\frac{A_{s2}}{Arb_{size} \cdot 2}\right)$ $N_{reqd} = 1$

Capacity of Supplied Reinforcing $N_{reqd} \cdot \left[Arb_{size} \cdot 2 \cdot \left(\phi_t \cdot f_y\right)\right] = 21.6 \cdot kip$







Width $W_{inTube} := 100 \cdot mm$ Thickness $t_{in} := 6 \cdot mm$

Height $H_{inTube} := 50 \cdot mm$ Height $d_{in} := H_{inTube}$

Tube Steel Yield Strength $F_{yts} := 50 \cdot ksi$

Eurocode Equations

Shear Capacity of Tube Steel

Ultimate Shear $F_v = 100 \cdot kN$ $F_v = 22.481 \cdot kip$

$\phi V_{ts} := f_{sd\_ts} \cdot 2 \cdot t_{in} \cdot \left(d_{in}\right)$ $\phi V_{ts} = 122.976 \cdot kN$ $\phi V_{ts} = 27.646 \cdot kip$

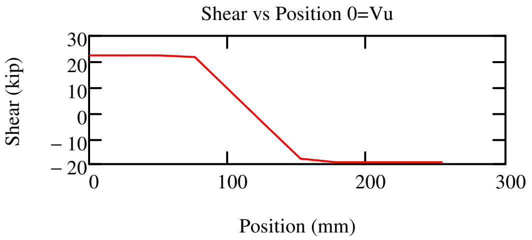
Moment Capacity of Tube Steel

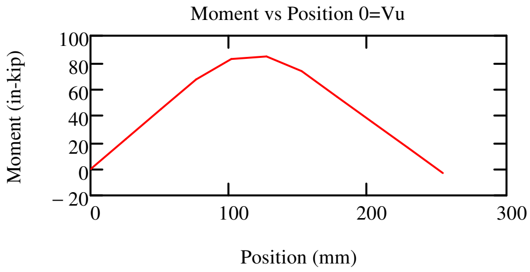
Ultimate Moment @ Location of zero shear

$ZeroShear := root\left(V_u(X), X\right)$ $ZeroShear = 4.654 \cdot in$ $M_{u\_zero} := \left|M_u(ZeroShear)\right|$ $M_{u\_zero} = 85.498 \cdot in \cdot kip$

Supplied Plastic Section Modulus $Z_{supplied} := 29200 \cdot mm^3$ $Z_{supplied} = 1.782 in^3$

$\phi M_p := f_{yd\_ts} \cdot Z_{supplied}$ $\phi M_p = 91.747 \cdot in \cdot kip$





US equivalent Equations

Shear Capacity of Tube Steel

Ultimate Shear

$F_v = 22.481 \cdot kip$ $\quad$ $F_v = 100 \cdot kN$ $\quad$ $F_v = 22.481 \cdot kip$

$\phi V_{ts} := 0.9 \cdot 0.6 \cdot F_{yts} \cdot 2 \cdot t_{in} \cdot \left(d_{in}\right)$ $\quad$ $\phi V_{ts} = 111.695 \cdot kN$ $\quad$ $\phi V_{ts} = 25.11 \cdot kip$

Moment Capacity of Tube Steel

Ultimate Moment @ Location of zero shear

$ZeroShear := root\left(V_u(X), X\right)$ $\quad$ $ZeroShear = 4.654 \cdot in$ $\quad$ $M_{u\_zero} := \left|M_u(ZeroShear)\right|$ $\quad$ $M_{u\_zero} = 85.498 \cdot in \cdot kip$

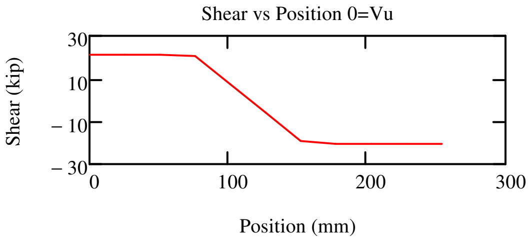
Supplied Plastic Section Modulus

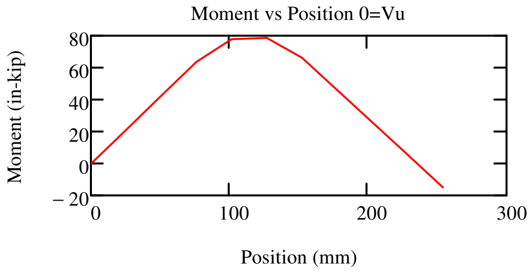
$Z_{supplied} = 1.782 in^3$

$\phi M_p := 0.9 \cdot F_{yts} \cdot Z_{supplied}$ $\quad$ $\phi M_p = 80.185 \cdot in \cdot kip$

Plastic Section Modulus not satisfied for US Code Equivalent, reduce Design Strength to

$F_v := 21.3 \cdot kip$





Moment Capacity of Tube Steel

$F_v = 94.747 \cdot kN$ $\quad$ $F_v = 21.3 \cdot kip$

Ultimate Moment @ Location of zero shear

$ZeroShear := root\left(V_u(X), X\right)$ $\quad$ $ZeroShear = 4.564 \cdot in$ $\quad$ $M_{u\_zero} := \left|M_u(ZeroShear)\right|$ $\quad$ $M_{u\_zero} = 80.056 \cdot in \cdot kip$

Supplied Plastic Section Modulus

$Z_{supplied} = 1.782 in^3$

$\phi M_p := 0.9 \cdot F_{yts} \cdot Z_{supplied}$ $\quad$ $\phi M_p = 80.185 \cdot in \cdot kip$