

MEMO 525

BSF - DESIGN OF REINFORCEMENT,  
CANTILEVERED BEAM-BEAM

DESIGN

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## BSF - DESIGN OF REINFORCEMENT, CANTILEVERED BEAM-BEAM

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## PART 1 BASIC ASSUMPTIONS

### 1.1 GENERAL

This memo deals with BSF used as beam-beam connections for continuous beams. Standard BSF-units and beam boxes are used. Reinforcement in the beam with the BSF knife is found in Memo 521. Therefore, only reinforcement related to the BSF beam-box is discussed.

As the cross sections of the two connected beams will vary, there may be issues with the local force transfer in the end of the beam that is not covered by the examples given in this Memo. **Therefore, the following calculations of anchorage of the units and the resulting reinforcement must be considered as an example to illustrate the calculation model.**

The EC-2 shall always be applied as the governing design document for the beam reinforcement. The information found here and in the memos assumes that the design of the elements and the use of the units in structural elements are carried out under the supervision of a structural engineer with knowledge about both the relevant standards, and the structural behaviour of concrete and steel structures.

### 1.2 STANDARDS

The calculations are in accordance with:

- Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for buildings.
- Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings.
- Eurocode 3: Design of steel structures. Part 1-8: Design of joints.

The selected values for the NDP's in the following calculations are:

Parameter	$\gamma_c$	$\gamma_s$	$\alpha_{cc}$	$\alpha_{ct}$
Value	1,5	1,15	0,85	0,85

Table 1: NDP-s in EC2.

Parameter	$\gamma_{M0}$	$\gamma_{M1}$	$\gamma_{M2}$
Value	1,1	1,1	1,25

Table 2: NDP-s in EC3.

## 1.3 QUALITIES

Concrete C35/45:  $f_{ck} = 35,0 \text{ MPa}$  EC2, Table 3.1  
 $f_{cd} = \alpha_{cc} \times f_{ck} / \gamma_c = 0,85 \times 35 / 1,5 = 19,8 \text{ MPa}$  EC2, Clause 3.15  
 $f_{ctd} = \alpha_{ct} \times f_{ctk,0,05} / \gamma_c = 0,85 \times 2,2 / 1,5 = 1,24 \text{ MPa}$  EC2, Clause 3.16  
 $f_{bd} = 2,25 \times \eta_1 \times \eta_2 \times f_{ctd} = 2,25 \times 1,0 \times 1,0 \times 1,24 = 2,79 \text{ MPa}$  EC2, Clause 8.4.2

*Note: For simplicity, good bond conditions are assumed when calculating  $f_{bd}$ . This assumption may not be correct in all situations and has to be evaluated in each case. EC2 indicates poor bond conditions for anchoring in top of the beam.*

Reinforcement 500C (EN 1992-1-1, Annex C):  $f_{yd} = f_{yk} / \gamma_s = 500 / 1,15 = 435 \text{ MPa}$  EC2, Clause 3.2.

*Note: Reinforcement steel of different qualities may be chosen provided that the calculations take into account the actual yield strength ( $f_y \leq 500 \text{ MPa}$ ) and that the bendability is sufficient for fitting the vertical suspension reinforcement to the half round steel.*

Steel Sxxx (EN 10025-2):

Steel S355: Tension:  $f_{yd} = f_y / \gamma_{M0} = 355 / 1,1 = 322 \text{ MPa}$   
 Compression:  $f_{yd} = f_y / \gamma_{M0} = 355 / 1,1 = 322 \text{ MPa}$   
 Shear:  $f_{sd} = f_y / (\gamma_{M0} \times \sqrt{3}) = 355 / (1,1 \times \sqrt{3}) = 186 \text{ MPa}$   
 Weld S355:  $f_{w,d} = \frac{f_u}{\gamma_{M2} \sqrt{3}} \times \frac{1}{\beta_w} = \frac{510}{1,25 \times \sqrt{3}} \times \frac{1}{0,9} = 262 \text{ MPa}$

Threaded bars/nut:

8.8 quality steel:  $f_{yd} = 0,9 \times f_u / \gamma_{M2} = 0,9 \times 800 / 1,25 = 576 \text{ MPa}$

## 1.4 DIMENSIONS AND CROSS-SECTION PARAMETERS

UNIT	HALF ROUND STEEL			HORIZONTAL ANCHORING <sup>1)</sup>	INTERNAL OPENING BEAM BOX (WIDTH×HEIGHT×DEPTH)
	D [mm]	L [mm]	Steel grade		
BSF225 BEAM BOX	Ø76	100	S355	2×M12, 8.8+ nut, L=650mm & st.pl.50×50×8, S355	30mm×215mm×80mm
BSF300 BEAM BOX	Ø76	100	S355	2×M12, 8.8+ nut, L=650mm & st.pl.50×50×8, S355	30mm×255mm×80mm
BSF450 BEAM BOX	Ø76	100	S355	1×M20, 8.8+ nut, L=750mm & st.pl.90×90×12, S355	40mm×270mm×92,5mm
BSF700 BEAM BOX	Ø175	140	S355	2×M20, 8.8+ nut, L=750mm & st.pl.160×90×12, S355	50mm×310mm×105mm

**Table 3: Dimensions– BSF beam box. <sup>1)</sup> See also Table 4. Note: The steel plate anchoring both the M20 bars for the BSF700 is designed only for the actual design force of 210kN, not the tensile capacity of two M20 bars.**

NOMINAL DIAMETER	M12	M16	M20
Equivalent diameter: $\varnothing_{eq}$ [mm]	10,4	14,1	17,7
Stress area: $A_s$ [mm <sup>2</sup> ]	84	157	245
Tensile capacity (8.8): $F_{cap} = f_{yd} \times A_s$ [kN]	48	90	141
With across flats: NV [mm]	19	24	30
Required dim. of square steel plate anchoring $F_{cap}$ : <sup>1</sup> $b_{req} \geq [F_{cap}/f_{cd} + \pi \times \varnothing_{nom}^2/4]^{0.5}$ [mm] Select b×b	≈50,4 Select 50×50	69 Select 70×70	86 Select 90×90
Net area for compression anchorage: $A_{net} = A_{steel\ plate} - \pi \times \varnothing_{nom}^2/4$ [mm <sup>2</sup> ]	2387	4699	7786
Concrete stress: $\sigma_c = F_{cap}/A_{net}$ [MPa]	20,1	19,1	18,1
Required thickness of steel plate, S355: <sup>1</sup> $a = (2^{0.5} \times b - NV)/2 \rightarrow t_1 \geq a \times (\sigma_c/f_{yd})^{0.5}$ [mm] $c = b/2 - NV/2 \rightarrow t_2 \geq 3^{0.5} \times c \times (\sigma_c/f_{yd})^{0.5}$ [mm] $t > [t_1, t_2]$	a=25,9 c=15,5 Select t=8mm	t <sub>1</sub> =6,5 t <sub>2</sub> =6,7 Select t=10mm	a=37,5 c=23 Select t=12mm
Standard height of nut: (H) [mm]	10,0	13,0	16,0
Required thread length in blind holes:	S355 18mm	24mm	30mm

**Table 4: Dimensions - threaded bars and anchoring steel plates.**

<sup>1</sup> An illustration, and background for the formulas, can be found in the Memo "BSF-Design of steel units". The listed dimensions are based on the concrete quality and parameters given in Section 1.2 and Section 1.3. Note: The steel plate anchoring both the M20 bars for the BSF700 is designed only for the actual design force of 210kN, not the tensile capacity of two M20 bars.

## 1.5 LOADS

Vertical ultimate limit state load:  $F_v$  = According to Table 5.

Horizontal ultimate limit state load - in axial direction:  $F_H=0\text{kN}$  (see notes below)

Horizontal ultimate limit state load - in transverse direction:  $F_T=0\text{kN}$

### \*Note on loads:

- The BSF beam box is a product designed to transfer primarily vertical load.
- Significant horizontal loading on the unit may also occur if imposed deformation (shrinkage, temperature differences etc.) in the pre-cast element is resisted. When the occurring horizontal force exceeds the potential friction force the knife will slide and the force will be partly relieved. The static friction factor steel-steel at support is assumed to be within the range (0,2-0,5). The maximum friction force due to gradually increasing imposed deformations will however be associated with vertical service loads. The steel parts of the unit, and anchoring of these parts into the concrete are designed for the following unfavourable load combination:

*Vertical force  $1,0F_v$  + Horizontal force  $0,3F_v$*

- In some cases transfer of static global horizontal load via the unit may be requested. The magnitude of this force would be limited by the minimum friction factor at the support and vertical load present at the same time. This will imply uncertainty in resistance, and it's recommended to transfer the horizontal forces by proper reinforcement through the joint. In case of dynamic loads, the horizontal resistance should always be assumed to be zero.
- Horizontal anchoring of the steel parts assumes minimum concrete grade C35 in column and beam.

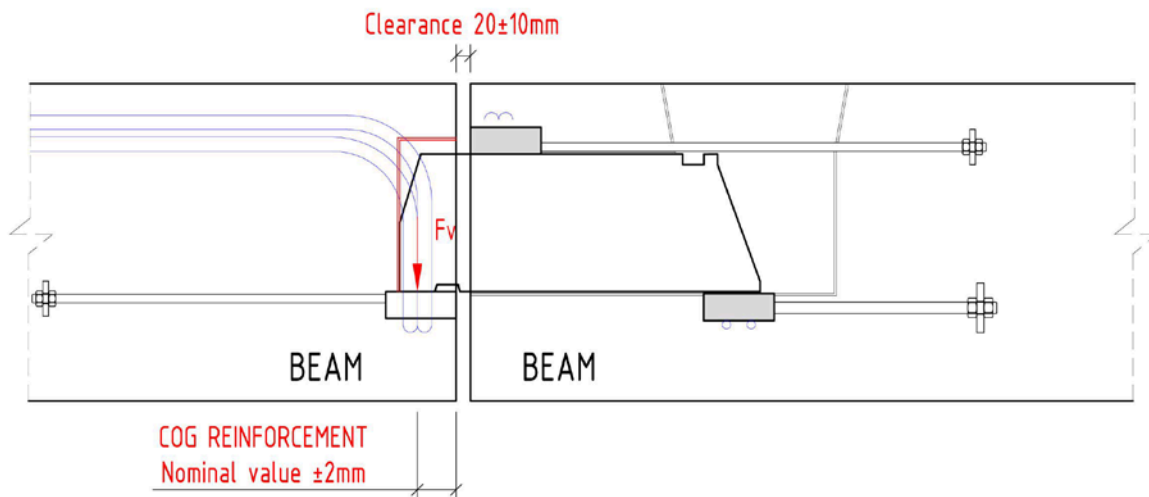
UNIT	VERTICAL ULTIMATE LIMIT STATE LOAD $F_v$ [kN]	LOAD BEAM BOX	
		VERT. $1,0F_v$ [kN]	HOR. $0,3F_v$ [kN]
BSF225	225	225	67,5
BSF300	300	300	90
BSF450	450	450	135
BSF700	700	700	210

**Table 5: Design loads**

## 1.6 TOLERANCES

The design nominal gap between two beams is 20mm, with a tolerance of  $\pm 10\text{mm}$ . The tolerances are handled with the cantilevering of the knife from the beam. If the gap is 30mm, the knife is pushed out an extra 10mm and vice versa if the gap is only 10mm. Thus, the load point in the beam box will always be the same. The knife shall always be pushed out until it bottoms against the back of the beam box.

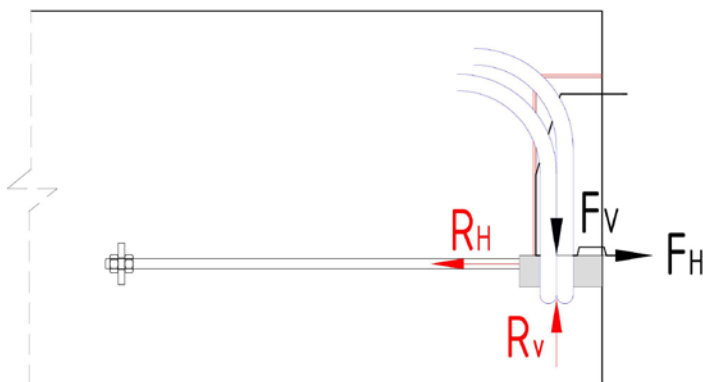
The tolerance on location of the reinforcement for the beam box is  $\pm 2\text{mm}$ .



**Figure 1: Tolerances. (cog= center of gravity)**

## **PART 2 PRINCIPAL DESIGN OF REINFORCEMENT - BSF BEAM BOX**

### **2.1 BEAM BOX - EQUILIBRIUM**



**Figure 2: Equilibrium.**

The assumed flow of forces is:

Vertical force:

Suspension reinforcement designed for the load is to be placed at the load point  $\Rightarrow R_v = F_v$ .

Horizontal force.

Anchored with threaded bars.  $\Rightarrow R_H = F_H$ . The bending moment associated with the small vertical shift in the horizontal force is neglected.



## 2.2 BEAM BOX – ANCORING REINFORCEMENT

### 1) Required cross section for suspension reinforcement:

$$A_s = \frac{F_v}{f_{yd}}$$

### 2) Mandrel diameter – Bending of reinforcement - EC2, clause 6.5.4:

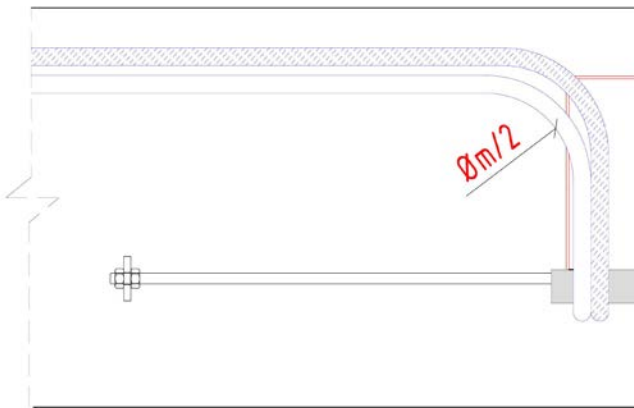


Figure 3: Bending of reinforcement.

Minimum mandrel diameter:

$$\varnothing_m = \frac{F_v}{b_{eff} \times 0,6 \times \left(1 - \frac{f_{ck}}{250}\right) \times f_{cd} \times 0,5}$$

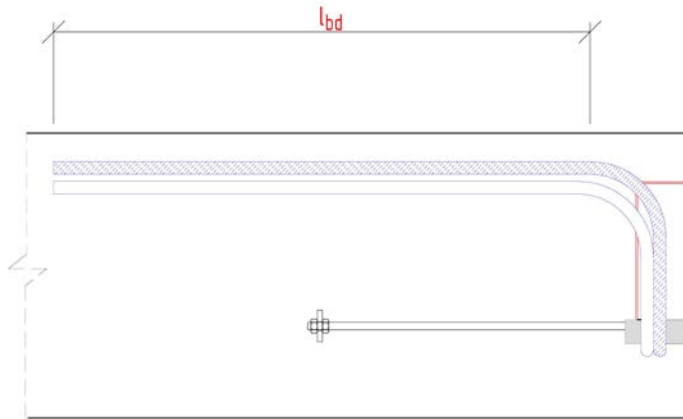
$b_{eff}$ = effective beam width. If the compression strut crosses the unit the width of the unit shall be extracted. Normally this will not be the case.

$\varnothing_m$ = Mandrel diameter of reinforcement

$\theta$  = Concrete strut assumed in 45degrees,  $\Rightarrow \sin\theta \times \cos\theta = 0,5$ , see also Memo 521, Part 2.

$\Rightarrow$  Select appropriate mandrel diameter. The minimum mandrel diameter shall comply with the requirements of EN 1992-1-1, 8.3.

### 3) Anchoring of reinforcement - EC2, clause 8.4.3 and 8.4.4:



**Figure 4: Anchoring of reinforcement.**

$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\sigma_{sd}}{f_{bd}} \times \frac{\sigma_{sd}}{f_{bd}}$$

$$\text{Stress in stirrup: } \sigma_{sd} = \frac{F_v}{A_s}$$

$A_s$  = Total area of selected reinforcement bars.

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \phi; 100\text{mm})$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \phi) / \phi$$

Neglecting any positive effect of concrete cover, selecting  $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting  $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 > 0,7$$

## 4) Lap of stirrups - EC2, clause 8.7.3:

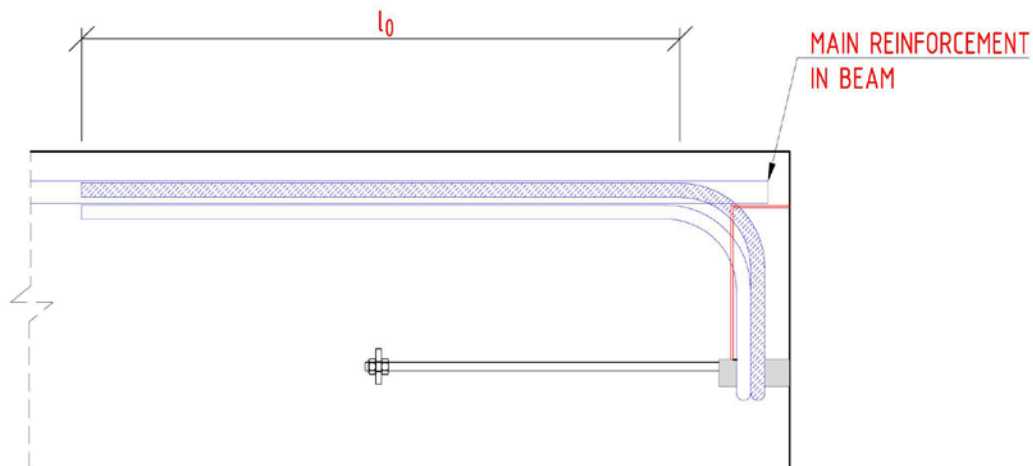


Figure 5: Lap of reinforcement.

$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, reqd} \geq l_{0, min}$$

Required lap length:

$l_{b, reqd}$  = as calculated in clause. 3.

$l_{0, min} = \max(0,3 \times \alpha_6 \times l_{b, reqd}; 15 \times \varnothing; 200 \text{ mm})$

Table 8.2:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_5 = 1,0$  as calculated in clause 3.

Table 8.3:  $\alpha_6 = 1.5$  (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times l_{b, reqd}$$

## 5) Anchoring of main reinforcement:

It must be ensured the horizontal part of the reinforcement is continued until the main reinforcement is sufficient anchored to carry the load.

## 2.3 BEAM BOX- HORIZONTAL ANCHORING

The beam box is anchored for a total horizontal load of  $F_H = 0,3F_V$ . The knife will be in contact with the half round steel and the horizontal force is transferred by friction between the two steel parts. The half round steel is anchored with threaded bars.

The required dimension of threaded bar and machined thread lengths in the half round steel is found from Table 4.

## 2.4 EVALUATION OF REINFORCEMENT IN THE END OF THE CONCRETE BEAM

### 2.4.1 STRUT AND TIE MODEL

The beam box will be located in the upper part of the beam cross section. Compatibility in strains through the cross section implies that some of the force will bypass the half round steel and spread into the underlying concrete. This is illustrated with a strut and tie model in Figure 6.

#### *Horizontal force in compression strut:*

The horizontal force in the assumed compression strut must be anchored with horizontal reinforcement inwards from the beam end. For design purpose, the horizontal force may be thought of as smeared, giving horizontal force intensity towards the vertical end of the beam:

For the case of  $z=2b$ , the horizontal force per unit height of the beam becomes:

$$1/2 \times F_v / (z/2) = F_v / z$$

For the case of  $z=3b$ , the horizontal force per unit height of the beam becomes:

$$1/3 \times F_v / (z/3) = F_v / z$$

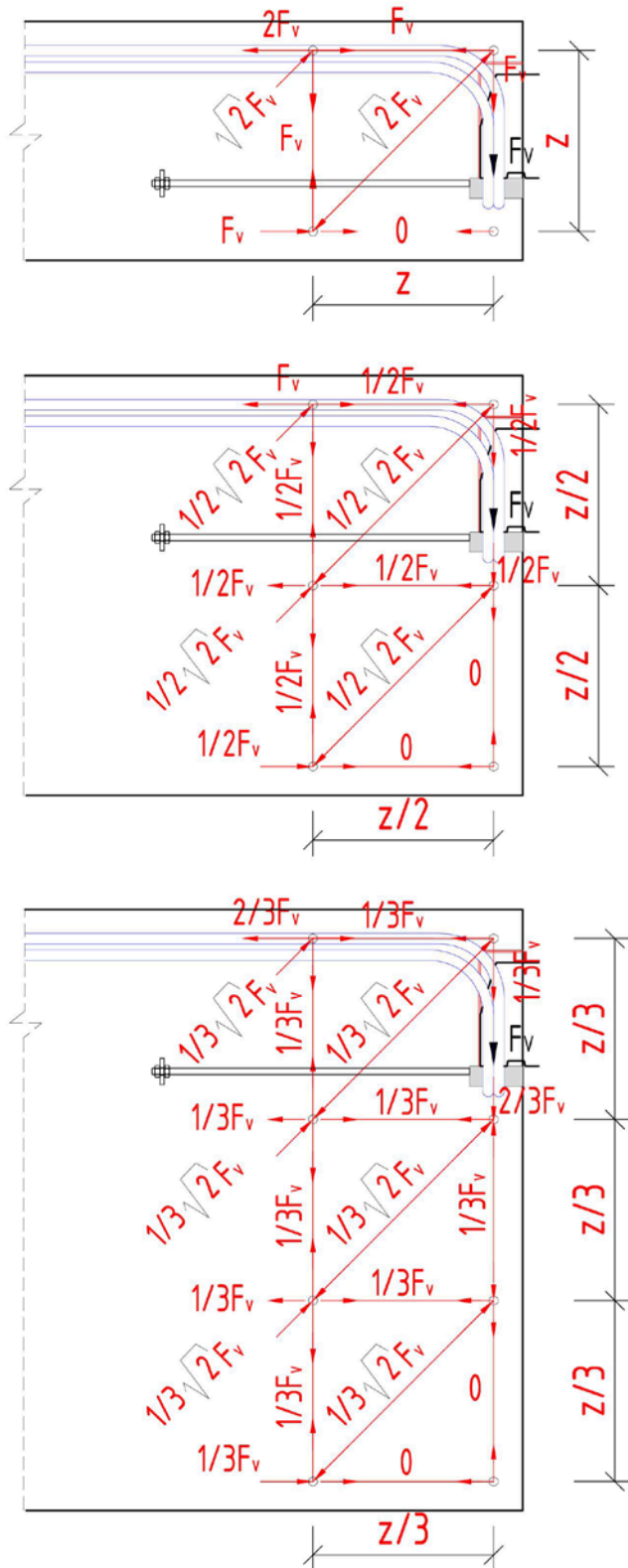
The above evaluation illustrates that the force intensity towards the end of the beam always becomes  $F_v/z$ . Thus, the intensity is depending on the beam height. Narrow stirrups (Just a bit wider than the half round steel) distributed just under the half round steel is recommended. It is important these stirrups are sufficient anchored inwards.

#### *Vertical force in compression strut:*

The vertical force in the compression strut will never exceed  $F_v$ . When the ordinary beam shear reinforcement (designed for the shear force  $F_v$ ) runs until the end of the beam, it will ensure integrity for the vertical force.

#### *Splitting force in transverse direction:*

Due to the shape of the half round steel, it is recommended always to include some reinforcement for splitting stress below the unit. This reinforcement may be designed according to EC2 clause. 6.5.3. Wide stirrups (as wide as the beam) distributed below the unit according to the recommendations may be applied.



**Figure 6: Strut and tie model in beam end. (Should be printed in colour)**

## 2.4.2 REINFORCEMENT IN TOP OF BEAM – BOND AND ANCHORING

As illustrated in Figure 7, drawing no. 1, the tension force at the top of the truss is “one ahead” of the compression force at the bottom of the truss. Proper anchoring of the reinforcement in top of the beam may conservatively be ensured at a distance  $z$  from the support. (This corresponds to a shift in the bending moment diagram a distance  $z$ ; see also EC2, clause 9.2.1.3 and clause 6.5.3. (7)).

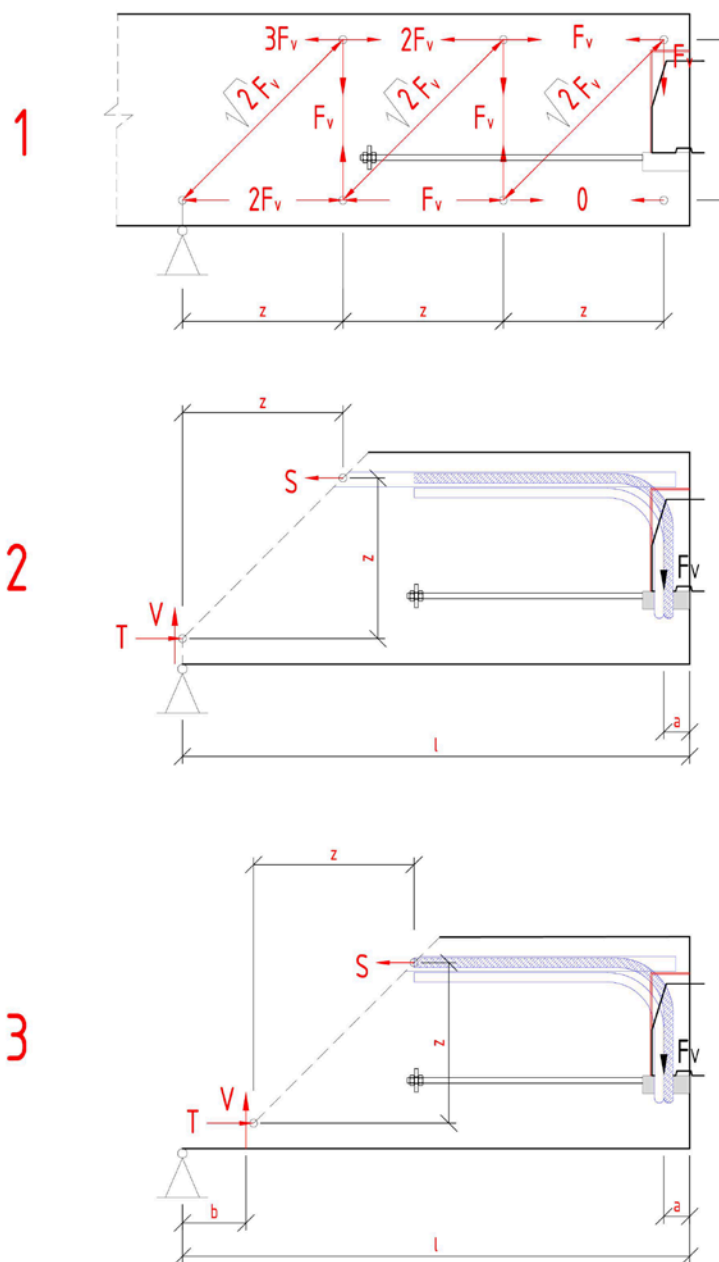


Figure 7: Illustration. (Should be printed in colour)

The tension in the reinforcement at distance  $z$  from the support equals the tension force at support. (Note: forces from other loads on the beam will come in addition):

$$S = \frac{F_v \times (l - a)}{z}$$

Estimate, required reinforcement:

$$A_s = S / f_{sd}$$

Anchoring length for top reinforcement (fully anchored):

$$l_n = \frac{\pi \times \varnothing^2 / 4 \times 435 \text{ MPa} \times n}{\pi \times \varnothing_n \times f_{bd}}$$

$\varnothing$  = diameter of bar

$\varnothing_n$  = diameter of bar. Equivalent diameter when bundled

$n$  = number of bars

$f_{bd}$  = bond stress

## Control 1: Anchoring at support:

Equivalent amount of fully anchored reinforcement:

$$A_{eqv} = A_{s,selected} \times (l - \text{concrete cover}) / l_n$$

$A_{s,selected}$  = Total amount of top reinforcement

$l_n$  = calculated anchoring length for fully anchored top reinforcement

Control:  $A_{eqv} > A_s$

(Anchored suspension reinforcement may also be added to  $A_{eqv}$ )

## Control 2: Anchoring at distance $z$ from support:

Equivalent amount of fully anchored reinforcement:

$$A_{eqv} = A_{s,selected} \times (l - z - \text{concrete cover}) / l_n$$

$A_{s,selected}$  = Total amount of top reinforcement

$l_n$  = calculated anchoring length for fully anchored top reinforcement

Control:  $A_{eqv} > A_s$

(Tension force to be anchored at distance  $z$  is equal to the tension force to be anchored at support.

Anchored suspension reinforcement may also be added to  $A_{eqv}$ )

## Control 3: Anchoring at the end of the suspension reinforcement:

This is relevant if the suspension reinforcement bars ends outside the distance  $z$  from the support, and if the suspension reinforcement is included in  $A_{eqv}$  in control 1 and 2. Control 3 is done in the same way as control 1&2. The tension is calculated for a situation as illustrated in drawing 3 in Figure 6.

$$S = \frac{F_v \times (l - a - b)}{z}$$

## Control 4: Bond/transfer of force into reinforcement at top of beam:

Sufficient bond in order to transfer the increase in tension along the beam into the top reinforcement must be ensured. This is a relevant issue for large concentrated cantilevered loads.

Increase in tension in the reinforcement per/mm:  $\Delta S(x)/dx = (\Delta M(x)/dx)/z = V(x)/z$

Capacity for increase in force by bond per/mm:  $\Delta S_{\text{bond}}/dx = f_{\text{bd}} \times \phi_n \times \pi \times n$

Control:  $\Delta S_{\text{bond}}(x)/dx > \Delta S(x)/dx$

### 2.4.3 SHEAR STIRRUPS IN BEAM END

The shear at the end of the beam equals  $F_V$ :

$$\frac{A_s}{s} = \frac{V_{Rd,s}}{z \times f_{yd}} = \frac{F_V}{z \times f_{yd}}$$

### 2.4.4 SHEAR COMPRESSION IN BEAM END

Shear compression: EC2, clause 6.2.3.

$$V_{Rd,max} = \alpha_{cw} \times b_w \times z \times u_1 \times f_{cd} / (\cot \theta + \tan \theta)$$

$$b_w = b_{\text{beam}}$$

### 2.4.5 HORIZONTAL BARS IN BEAM END

Narrow stirrups for the horizontal force according to strut and tie model:

$$\frac{A_s}{s} = \frac{F_V}{z \times f_{yd}}$$

A total cross section area equal to:  $A_s/s \times H$ , shall be included. (Both legs on the stirrups is active)

Wide stirrups for splitting stress, EC2, clause. 6.5.3:

If  $b < H/2$ :

$$A_s = \frac{1}{4} \times \frac{b-a}{b} \times F_v \times \frac{1}{f_{yd}}$$

If  $b > H/2$ :

$$A_s = \frac{1}{4} \times \left(1 - 0,7 \frac{a}{H/2}\right) \times F_v \times \frac{1}{f_{yd}}$$

⇒ Conservative simplification:

$$A_s = \frac{1}{4} \times F_v \times \frac{1}{f_{yd}}$$

(Only one leg per stirrup is active in transverse direction)



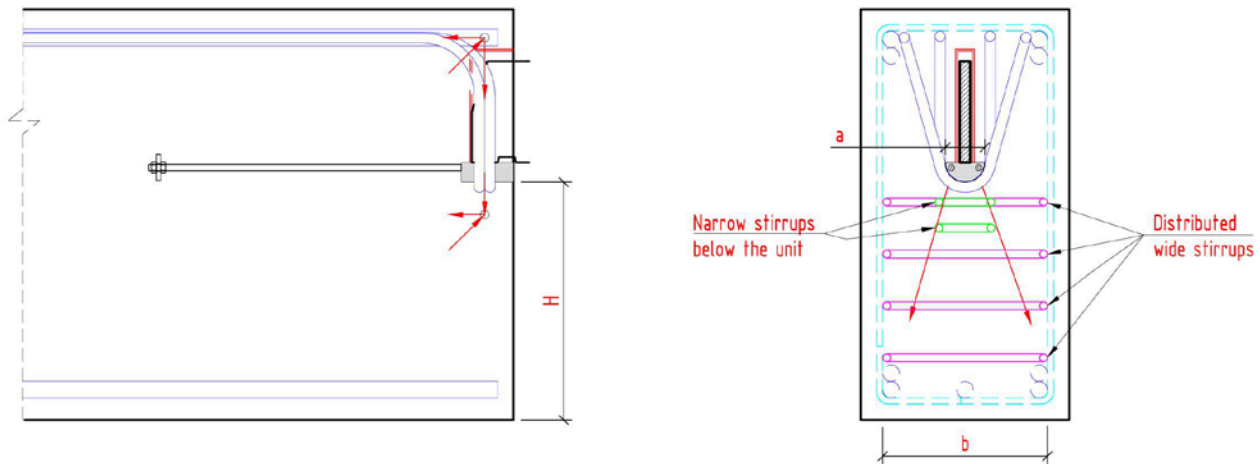


Figure 8: Illustration – horizontal stirrups in beam end.

## PART 3 – BSF 225

### 3.1 BEAM BOX – ANCHORING REINFORCEMENT

(Note: In the example calculations, «good» bond conditions are assumed when calculating  $f_{bd}$ . This may not be the case at the top of the beam, see EC2, clause 8.4.2 (2))

#### 1) Required cross section for reinforcement:

$$A_s = \frac{F_V}{f_{yd}} = \frac{225kN}{435MPa} = 517mm^2$$

$$2\emptyset 16 \text{ stirrups} = 201mm^2 \times 4 = 804mm^2$$

$$\text{Capacity of selected reinforcement: } 804mm^2 \times 435MPa = 349kN$$

#### 2) Mandrel diameter – Bending of reinforcement - EC2, clause 6.5.4:

$$\emptyset_{mf, \min} = \frac{F_V}{b_{eff} \times 0,6 \times (1 - \frac{f_{ck}}{250}) \times f_{cd} \times 0,5} = \frac{225000}{300 \times 0,6 \times (1 - \frac{35}{250}) \times 19,8MPa \times 0,5} = 147 \text{ mm}$$

$b_{eff}$  = effective width of beam. Assume:  $b_{eff} = b_{beam} = 300mm$

$\emptyset_{mf}$  = Mandrel diameter of reinforcement.

Concrete strut assumed in 45degrees, se Part 2.

⇒ Select:  $\varnothing=200\text{mm}$

### 3) Anchoring of reinforcement, EC2 clause 8.4.3 and 8.4.4:

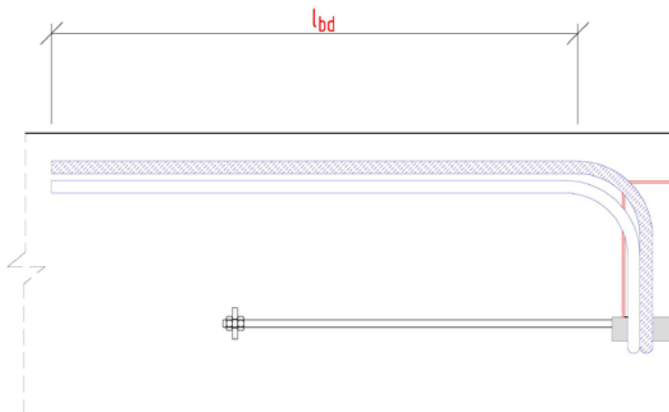


Figure 9: Anchoring of reinforcement.

$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\varnothing}{4} \times \frac{\sigma_{sd}}{f_{bd}}$$

$$\text{Stress in reinforcement: } \sigma_{sd} = \frac{225\text{kN}}{804\text{mm}^2} = 280\text{MPa}$$

$$l_{b,reqd} = \frac{16}{4} \times \frac{280}{2,79} = 401\text{mm}$$

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \varnothing; 100\text{mm}) = 160\text{mm}$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \varnothing) / \varnothing$$

Neglecting any positive effect of concrete cover, selecting  $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting  $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

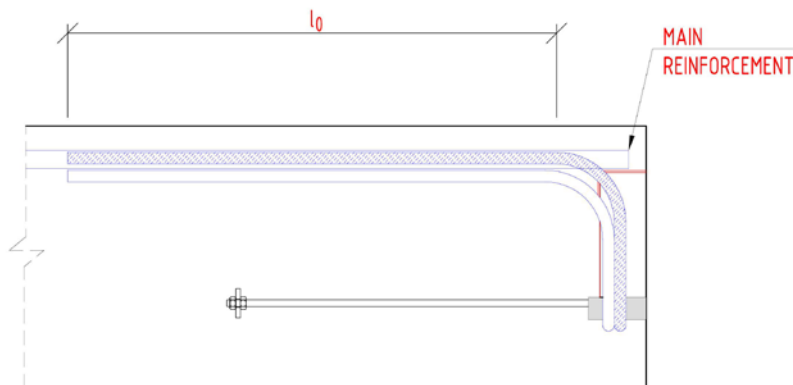
$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$

$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 401 \text{ mm} = 401 \text{ mm}$$

**4) Lap of stirrups, EC2 clause 8.7.3:**



**Figure 10: Lap of reinforcement.**

$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, \text{reqd}} \geq l_{0, \text{min}}$$

Required lap length:

$$l_{b, \text{reqd}} = 401 \text{ mm, see evaluation in clause 3.}$$

$$l_{0, \text{min}} = \max(0,3 \times \alpha_6 \times l_{b, \text{reqd}}; 15 \times \varnothing; 200 \text{ mm})$$

Table 8.2:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_5 = 1,0$  as calculated in clause 3.

Table 8.3:  $\alpha_6 = 1.5$  (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 401 \text{ mm} = 602 \text{ mm}$$

$$\Rightarrow \text{Select: } l_0 = 600 \text{ mm}$$

## 3.2 BEAM BOX – HORIZONTAL ANCHORING

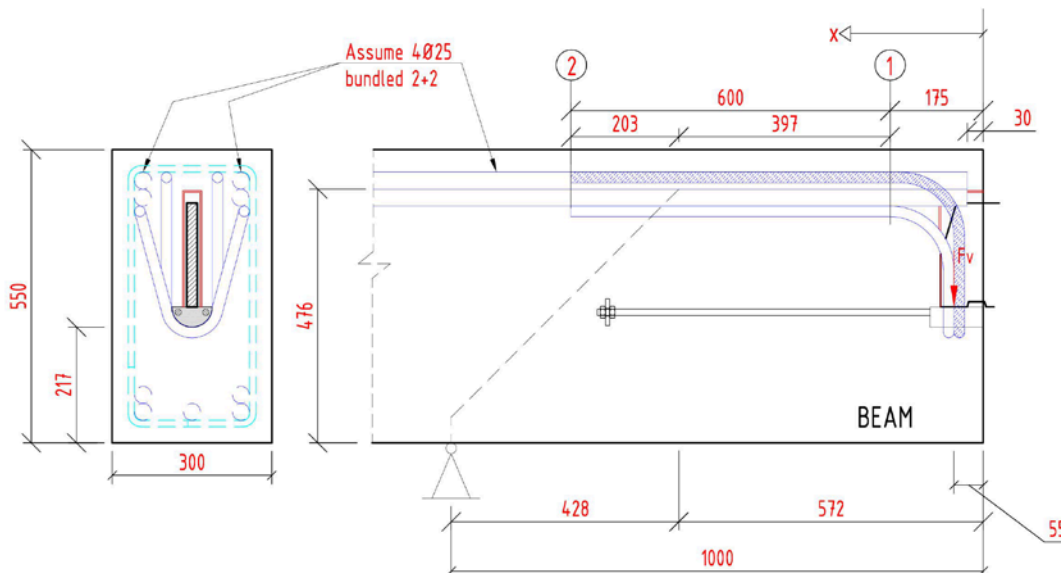
Horizontal anchoring of half round steel:  $R_H = 0,3 \times F_V = 67,5 \text{ kN}$ :

Select: 2xM12 threaded bars, 8.8 with nut & steel plate =  $48 \text{ kN} \times 2 = 96 \text{ kN}$

### 3.3 EXAMPLE – REINFORCEMENT IN BEAM END

Assume:

- Columns with five meters spacing. Beam-beam connection at 1m cantilevering from column.
- Cross section as illustrated in Figure 11.
- $z=0,9 \times d=0,9 \times 476\text{mm}=428\text{mm}$
- Horizontal part of the suspension reinforcement is 600mm ( $\approx$  equals the minimum calculated lap length). I.e. the bars end at  $x=175+600=775\text{mm}$ . (The final required length is found from the calculations)
- Neglecting self-weight. Assumed dead and live loads = 0kN/m



**Figure 11: Example – Beam with BSF225 beam box. (Note, the illustrated reinforcement does not represent the conclusion from the evaluations, follow the calculations below.)**

#### 3.3.1 REINFORCEMENT IN TOP OF BEAM– BOND AND ANCHORING

The tensile force in the reinforcement at top of the beam at distance  $z$  from the support:

$$S = \frac{225\text{kN} \times (1000 - 55)\text{mm}}{428\text{mm}} = 496\text{kN}$$

Estimate, required reinforcement:

$$A_s = 496\text{kN} / 435\text{MPa} = 1141\text{mm}^2$$

$\Rightarrow$  Assume main reinforcement at top of beam: 4Ø25 bundled 2+2 ( $\approx 1963\text{mm}^2$ )

Equivalent diameter of 2Ø25 bundled:

$$\phi_n = \phi \times \sqrt{2} = 25 \times \sqrt{2} = 35mm$$

Anchoring length of a bundle:

$$L_n = \frac{\pi \times 12,5^2 \times 435MPa \times 2}{\pi \times \phi_n \times f_{bd}} = \frac{\pi \times 12,5^2 \times 435MPa \times 2}{\pi \times 35 \times 2,79MPa} = \frac{427kN}{0,3067kN / mm} = 1392mm$$

Control 1: Anchoring at support (x=1000mm):

Equivalent fully anchored reinforcement:

$$A_{eqv} = 1963mm^2 / 1392mm \times (1000-30)mm = 1367mm^2$$

$$A_{eqv} > 1141mm^2 \Rightarrow OK.$$

Control 2: Anchoring at distance z from support (x=1000-428=572mm).

Equivalent fully anchored reinforcement:

$$A_{eqv} = 1963mm^2 \times (572-30)mm / 1392mm = 764mm^2$$

$$A_{eqv} < 1141mm^2 \Rightarrow NOT OK.$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in  $\phi 25$ :

$$S_{\phi 25} = 854kN / 1392mm \times (572-30)mm = 332kN$$

Not anchored:  $\Delta S = 496kN - 332kN = 164kN$

Required anchoring length 4 $\phi 16$ :

$$L_n = \frac{164000N}{\pi \times \phi \times f_{bd} \times 4} = \frac{164000N}{\pi \times 16 \times 2,79MPa \times 4} = 292mm$$

Transfer of force to the main reinforcement with lap of bars. Select  $l_0 = 1,5 \times l_n = 1,5 \times 292mm = 438mm$

Available length:  $L_{\phi 16} = 203mm$ , see Figure 14.

$\Rightarrow$  Solution: Horizontal part of suspension reinforcement is elongated 300mm.

Control 3: Anchoring at the end of the suspension reinforcement (x=775mm):

In the example, this point is within a distance z from the support. Thus, the tension and the required reinforcement will be as calculated in control 1:

Equivalent fully anchored reinforcement:

$$A_{eqv} = 1963mm^2 \times (775-30)mm / 1392mm = 1045mm^2$$

$$A_{eqv} < 1141mm^2 \Rightarrow NOT OK.$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in  $\phi 25$ :

$$S_{\phi 25} = A_{eqv} \times 435MPa = 1045mm^2 \times 435MPa = 455kN$$

Not anchored:  $\Delta S = 496kN - 455kN = 41kN$

Required anchoring length 4 $\phi 16$ :

$$L_n = \frac{41000N}{\pi \times \emptyset \times f_{bd} \times 4} = \frac{41000N}{\pi \times 16 \times 2,79MPa \times 4} = 73mm$$

⇒ This is ok if the horizontal part of the suspension reinforcement is elongated 300mm as stated in control 2.

**Control 4: Bond/transfer of force into reinforcement at top of beam:**

Increase in force per/mm:

$$\Delta S(x)/dx = (\Delta M(x)/dx)/z = V(x)/z = 225kN/428mm = 525N/mm$$

Capacity for increase in force by bond per/mm:

$$\Delta S_{bond}(x)/dx = f_{bd} \times \emptyset_n \times \pi \times 2 = 2,79 \times 35 \times \pi \times 2 = 613N/mm$$

$\Delta S_{bond}(x)/dx > \Delta S(x)/dx \Rightarrow$  OK. The bond to the main reinforcement is sufficient to take the increase in force.

### 3.3.2 SHEAR STIRRUPS IN BEAM END

Use a strut-and-tie model with compression diagonal at 45°. The shear at the end of the beam is  $F_V = 225kN$ . Beam as illustrated in Figure 11.

$$\frac{A_s}{s} = \frac{V_{Rd,s}}{z \times f_{yd}} \approx \frac{225 \times 10^3 N}{0,428m \times 435MPa} = 1209mm^2 / m$$

Assume stirrup diameter  $\emptyset 10$

⇒ Select  $\emptyset 10c/c100$  ( $1570mm^2/m$ )

### 3.3.3 SHEAR COMPRESSION IN BEAM END

Shear compression: EC2, clause 6.2.3. Beam as illustrated in Figure 11.

$$V_{Rd,max} = \alpha_{cw} \times b_w \times z \times u_1 \times f_{cd} / (\cot \theta + \tan \theta)$$

$$b_w = b_{beam} = 300mm$$

$$V_{Rd,max} = \{1,0 \times 300 \times 428 \times 0,6 \times [1 - (35/250)] \times 19,8 / (1+1)\} \times 10^{-3}$$

$$V_{Rd,max} = 655 kN (> V_{Rd} \Rightarrow OK)$$

### 3.3.4 HORIZONTAL BARS IN BEAM END

Example: Beam as illustrated in Figure 11:

Narrow stirrups for horizontal force:

Assume  $z=0,9 \times d$

$$\frac{A_s}{s} \times h = \frac{F_v}{z \times f_{yd}} \times h = \frac{225000N}{0,9 \times 428mm \times 435MPa} \times 217mm = 291mm^2$$

Select two narrow u-bars:  $\emptyset 12 = \pi \times 6^2 \times 4 = 452mm^2$ . Placed just below the unit.

Simplified: Horizontal length of bar:  $L = (z - H) + 40\emptyset = (476 - 217)mm + 40 \times 12mm \approx 800mm$

Wide stirrups for splitting force:

$$A_s = \frac{1}{4} \times \frac{F_v}{f_{yd}} = \frac{1}{4} \times \frac{225000N}{435MPa} = 130mm^2$$

Select two u-bars:  $\emptyset 12 = \pi \times 6^2 \times 2 = 226mm^2$ . Distributed below the unit.

Simplified: Horizontal length of bar:  $L = 40\emptyset = 40 \times 12mm \approx 500mm$ .

## PART 4 - BSF 300

### 4.1 BEAM BOX – ANCHORING REINFORCEMENT

(Note: In the example calculations, «good» bond conditions are assumed when calculating  $f_{bd}$ . This may not be the case at the top of the beam, see EC2, clause 8.4.2 (2))

#### 1) Required cross section for reinforcement:

$$A_s = \frac{F_v}{f_{yd}} = \frac{300kN}{435MPa} = 689mm^2$$

$$2\emptyset 16 \text{ stirrups} = 201mm^2 \times 4 = 804mm^2$$

$$\text{Capacity of selected reinforcement: } 804mm^2 \times 435MPa = 349kN$$

#### 2) Mandrel diameter – Bending of reinforcement - EC2, clause 6.5.4:

$$\emptyset_{mf, \min} = \frac{F_v}{b_{eff} \times 0,6 \times \left(1 - \frac{f_{ck}}{250}\right) \times f_{cd} \times 0,5} = \frac{300000}{300 \times 0,6 \times \left(1 - \frac{35}{250}\right) \times 19,8MPa \times 0,5} = 195 \text{ mm}$$

$b_{eff}$  = effective width of beam. Assume:  $b_{eff} = b_{beam} = 300mm$

$\emptyset_{mf}$  = Mandrel diameter of reinforcement.

Concrete strut assumed in 45degrees, se Part 2.

⇒ Select:  $\emptyset = 200mm$

### 3) Anchoring of reinforcement, EC2 clause 8.4.3 and 8.4.4:

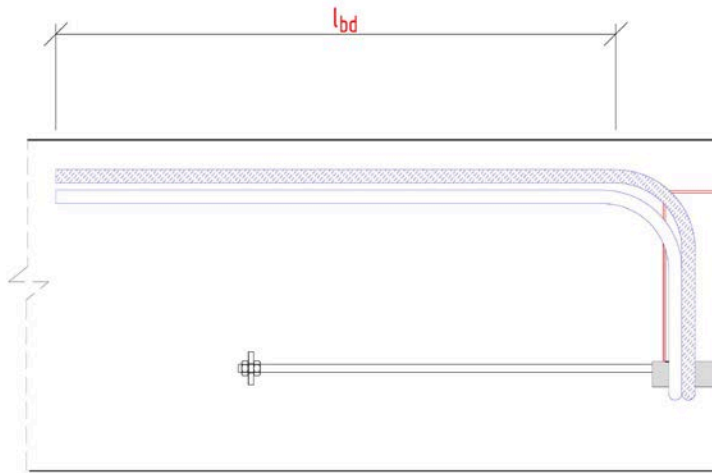


Figure 12: Anchoring of reinforcement.

$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\sigma_{sd}}{f_{bd}} \times \frac{\phi}{4}$$

$$\text{Stress in reinforcement: } \sigma_{sd} = \frac{300kN}{804mm^2} = 373MPa$$

$$l_{b,reqd} = \frac{16}{4} \times \frac{373}{2,79} = 535mm$$

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \phi; 100mm) = 160mm$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \phi) / \phi$$

Neglecting any positive effect of concrete cover, selecting  $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting  $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

$$\alpha_5 = 1,0$$

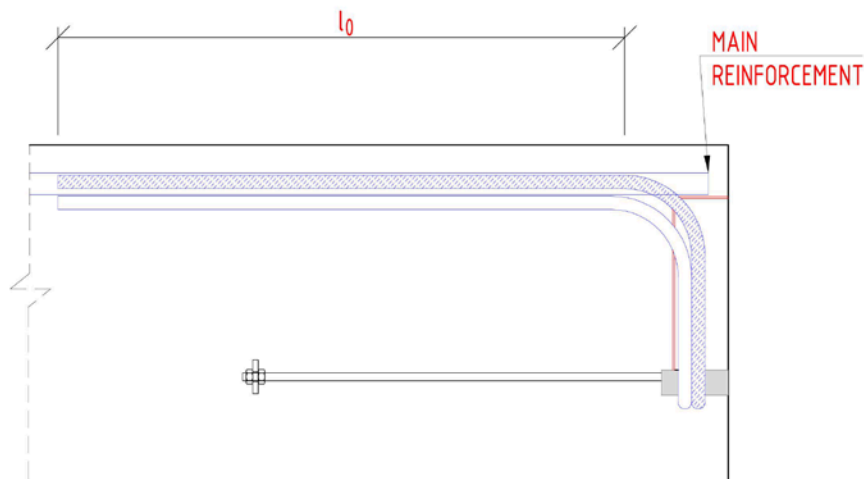
Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$



$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 535 \text{ mm} = 535 \text{ mm}$$

**4) Lap of stirrups, EC2 clause 8.7.3:**



**Figure 13: Lap of reinforcement.**

$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, \text{reqd}} \geq l_{0, \text{min}}$$

Required lap length:

$$l_{b, \text{reqd}} = 535 \text{ mm, see evaluation in clause 3.}$$

$$l_{0, \text{min}} = \max(0,3 \times \alpha_6 \times l_{b, \text{reqd}}; 15 \times \varnothing; 200 \text{ mm})$$

Table 8.2:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_5 = 1,0$  as calculated in clause 3.

Table 8.3:  $\alpha_6 = 1.5$  (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 535 \text{ mm} = 802 \text{ mm}$$

$\Rightarrow$  Select:  $l_0 = 800 \text{ mm}$

## 4.2 BEAM BOX – HORIZONTAL ANCHORING

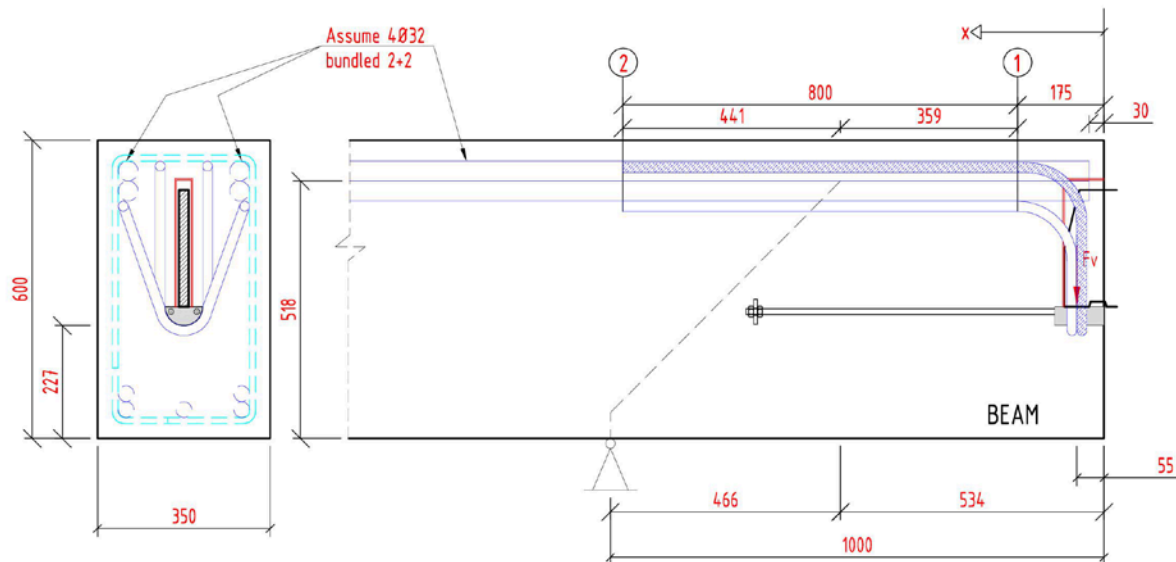
Horizontal anchoring of half round steel:  $R_H = 0,3 \times F_V = 90 \text{ kN}$ :

Select: 2×M12 threaded bars, 8.8 with nut & steel plate =  $48 \text{ kN} \times 2 = 96 \text{ kN}$

### 4.3 EXAMPLE – REINFORCEMENT IN BEAM END

Assume:

- Columns with 4,8m spacing. Beam-beam connection at 1m cantilevering from column.
- Cross section as illustrated in Figure 14.
- $z=0,9 \times d=0,9 \times 518\text{mm}=466\text{mm}$
- Horizontal part of the suspension reinforcement is 800mm ( $\approx$  equals the minimum calculated lap length). I.e. the bars end at:  $x=175+800=975\text{mm}$ . (The final required length is found from the calculations)
- Neglecting self-weight. Assumed dead and live loads = 0kN/m



**Figure 14: Example – Beam with BSF300 beam box. (Note, the illustrated reinforcement does not represent the conclusion from the evaluations, follow the calculations below.)**

#### 4.3.1 REINFORCEMENT IN TOP OF BEAM– BOND AND ANCHORAGE

The tensile force in the reinforcement at top of the beam at distance  $z$  from the support:

$$S = \frac{300\text{kN} \times (1000 - 55)\text{mm}}{466\text{mm}} = 608\text{kN}$$

Estimate, required reinforcement:

$$A_s = 608\text{kN} / 435\text{MPa} = 1398\text{mm}^2$$

$\Rightarrow$  Assume main reinforcement at top of beam: 4 $\emptyset$ 32 bundled 2+2 (=3216mm<sup>2</sup>)

Equivalent diameter of 2 $\emptyset$ 32 bundled:

$$\phi_n = \phi \times \sqrt{2} = 32 \times \sqrt{2} = 45\text{mm}$$

Anchoring length of a bundle:

$$L_n = \frac{\pi \times 16^2 \times 435\text{MPa} \times 2}{\pi \times \phi_n \times f_{bd}} = \frac{\pi \times 16^2 \times 435\text{MPa} \times 2}{\pi \times 45 \times 2,79\text{MPa}} = \frac{700\text{kN}}{0,3944\text{kN / mm}} = 1774\text{mm}$$

Control 1: Anchoring at support (x=1000mm):

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (1000 - 30)\text{mm} / 1774\text{mm} = 1758\text{mm}^2$$

$$A_{eqv} > 1398\text{mm}^2 \Rightarrow \text{OK.}$$

Control 2: Anchoring at distance z from support (x=1000-466=534mm).

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (534 - 30)\text{mm} / 1774\text{mm} = 914\text{mm}^2$$

$$A_{eqv} < 1398\text{mm}^2 \Rightarrow \text{NOT OK.}$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in  $\phi 32$ :

$$S_{\phi 32} = A_{eqv} \times 435\text{MPa} = 914\text{mm}^2 \times 435\text{MPa} = 398\text{kN}$$

$$\text{Not anchored: } \Delta S = 608\text{kN} - 398\text{kN} = 210\text{kN}$$

Required anchoring length  $4\phi 16$ :

$$L_n = \frac{210000\text{N}}{\pi \times \phi \times f_{bd} \times 4} = \frac{210000\text{N}}{\pi \times 16 \times 2,79\text{MPa} \times 4} = 374\text{mm}$$

Transfer of force to the main reinforcement with lap of bars. Select  $l_0 = 1,5 \times l_n = 1,5 \times 374\text{mm} = 561\text{mm}$

Available length:  $L_{\phi 16} = 441\text{mm}$ , see Figure 14.

$\Rightarrow$  Solution: Horizontal part of suspension reinforcement is elongated 200mm.

Control 3: Anchoring at the end of the suspension reinforcement (x=975mm):

In the example, this point is close to the support. Thus, the tension and the required reinforcement will be as calculated in control 1:

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (975 - 30)\text{mm} / 1774\text{mm} = 1713\text{mm}^2$$

$$A_{eqv} > 1398\text{mm}^2 \Rightarrow \text{OK.}$$

#### Control 4: Bond/transfer of force into reinforcement at top of beam:

Increase in force per/mm:

$$\Delta S(x)/dx = (\Delta M(x)/dx)/z = V(x)/z = 300\text{kN}/466\text{mm} = 644\text{N/mm}$$

Capacity for increase in force by bond per/mm:

$$\Delta S_{\text{bond}}(x)/dx = f_{\text{bd}} \times \phi_n \times \pi \times 2 = 2,79 \times 45 \times \pi \times 2 = 788\text{N/mm}$$

$\Delta S_{\text{bond}}(x)/dx > \Delta S(x)/dx \Rightarrow \text{OK}$ . The bond to the main reinforcement is sufficient to take the increase in force.

#### **4.3.2 SHEAR STIRRUPS IN BEAM END**

Use a strut-and-tie model with compression diagonal at 45°. The shear at the end of the beam is  $F_V = 300\text{kN}$ . Beam as illustrated in Figure 14.

$$\frac{A_s}{s} = \frac{V_{Rd,s}}{z \times f_{yd}} \approx \frac{300 \times 10^3 \text{ N}}{0,466\text{m} \times 435\text{MPa}} = 1480\text{mm}^2 / \text{m}$$

Assume stirrup diameter  $\phi 12 \Rightarrow \text{Select } \phi 12\text{c}/\text{c}100 \text{ (} 2261\text{mm}^2/\text{m} \text{)}$

#### **4.3.3 SHEAR COMPRESSION IN BEAM END**

Shear compression: EC2, clause 6.2.3. Beam as illustrated in Figure 14.

$$V_{Rd,\text{max}} = \alpha_{cw} \times b_w \times z \times u_1 \times f_{cd} / (\cot \theta + \tan \theta)$$

$$b_w = b_{\text{beam}} = 350\text{mm}$$

$$V_{Rd,\text{max}} = \{1,0 \times 350 \times 466 \times 0,6 \times [1 - (35/250)] \times 19,8 / (1+1)\} \times 10^{-3}$$

$$V_{Rd,\text{max}} = 833 \text{ kN } (> V_{Rd} \Rightarrow \text{OK})$$

#### **4.3.4 HORIZONTAL BARS IN BEAM END**

Example: Beam as illustrated in Figure 14:

Narrow stirrups for horizontal force:

Assume  $z = 0,9 \times d$

$$\frac{A_s}{s} \times h = \frac{F_V}{z \times f_{yd}} \times h = \frac{300000\text{N}}{0,9 \times 518\text{mm} \times 435\text{MPa}} \times 227\text{mm} = 336\text{mm}^2$$

Select two narrow u-bars:  $\phi 12 = \pi \times 6^2 \times 4 = 452\text{mm}^2$ . Placed just below the unit.

Simplified: Horizontal length of bar:  $L = (z - H) + 40\phi = (518 - 227)\text{mm} + 40 \times 12\text{mm} \approx 800\text{mm}$

Wide stirrups for splitting force:

$$A_s = \frac{1}{4} \times \frac{F_v}{f_{yd}} = \frac{1}{4} \times \frac{300000N}{435MPa} = 172mm^2$$

Select two u-bars:  $\emptyset 12 = \pi \times 6^2 \times 2 = 226mm^2$ . Distributed below the unit.

Simplified: Horizontal length of bar:  $L = 40\emptyset = 40 \times 12mm \approx 500mm$ .

## PART 5 – BSF 450

### 5.1 BEAM BOX – ANCHORING REINFORCEMENT

(Note: In the example calculations, «good» bond conditions are assumed when calculating  $f_{bd}$ . This may not be the case at the top of the beam, see EC2, clause 8.4.2 (2))

#### 1) Required cross section for reinforcement:

$$A_s = \frac{F_v}{f_{yd}} = \frac{450kN}{435MPa} = 1035mm^2$$

$$3\emptyset 16 \text{ stirrups} = 201mm^2 \times 6 = 1206mm^2$$

$$\text{Capacity of selected reinforcement: } 1206mm^2 \times 435MPa = 524kN$$

#### 2) Mandrel diameter – Bending of reinforcement - EC2, clause 6.5.4:

$$\emptyset_{mf, \min} = \frac{F_v}{b_{eff} \times 0,6 \times (1 - \frac{f_{ck}}{250}) \times f_{cd} \times 0,5} = \frac{450000}{350 \times 0,6 \times (1 - \frac{35}{250}) \times 19,8MPa \times 0,5} = 251 \text{ mm}$$

$b_{eff}$  = effective width of beam. Assume:  $b_{eff} = b_{beam} = 350mm$

$\emptyset_{mf}$  = Mandrel diameter of reinforcement.

Concrete strut assumed in 45degrees, se Part 2.

⇒ Select:  $\emptyset = 320mm$

### 3) Anchoring of reinforcement, EC2 clause 8.4.3 and 8.4.4:

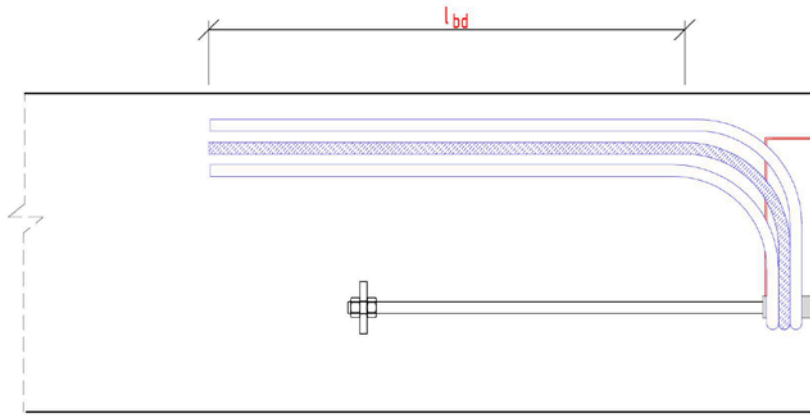


Figure 15: Anchoring of reinforcement.

$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\sigma_{sd}}{f_{bd}} \times \frac{\sigma_{sd}}{4}$$

$$\text{Stress in reinforcement: } \sigma_{sd} = \frac{450kN}{1206mm^2} = 373MPa$$

$$l_{b,reqd} = \frac{16}{4} \times \frac{373}{2,79} = 535mm$$

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \phi; 100mm) = 160mm$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \phi) / \phi$$

Neglecting any positive effect of concrete cover, selecting  $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting  $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

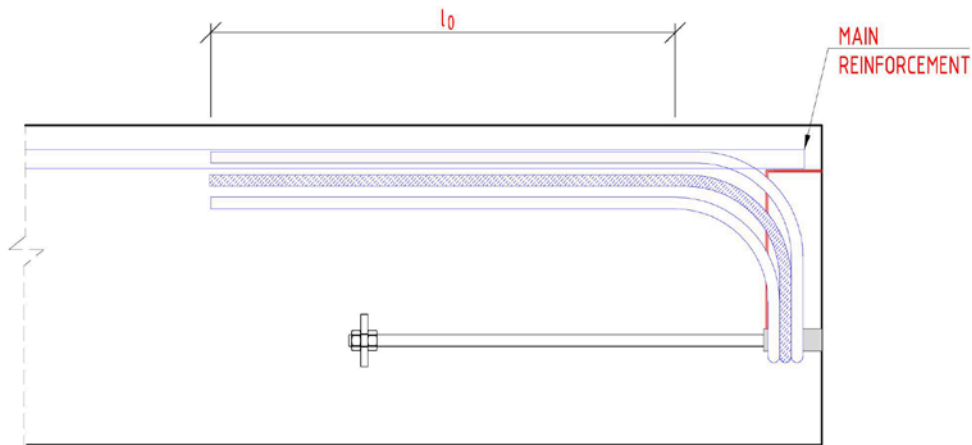
$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$

$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,0 \times 535 \text{ mm} = 535 \text{ mm}$$

**4) Lap of stirrups, EC2 clause 8.7.3:**



**Figure 16: Lap of reinforcement.**

$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, \text{reqd}} \geq l_{0, \text{min}}$$

Required lap length:

$$l_{b, \text{reqd}} = 535 \text{ mm, see evaluation in clause 3.}$$

$$l_{0, \text{min}} = \max(0,3 \times \alpha_6 \times l_{b, \text{reqd}}; 15 \times \varnothing; 200 \text{ mm})$$

Table 8.2:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_5 = 1,0$  as calculated in clause 3.

Table 8.3:  $\alpha_6 = 1.5$  (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 535 \text{ mm} = 802 \text{ mm}$$

$$\Rightarrow \text{Select: } l_0 = 800 \text{ mm}$$

## 5.2 BEAM BOX – HORIZONTAL ANCHORING

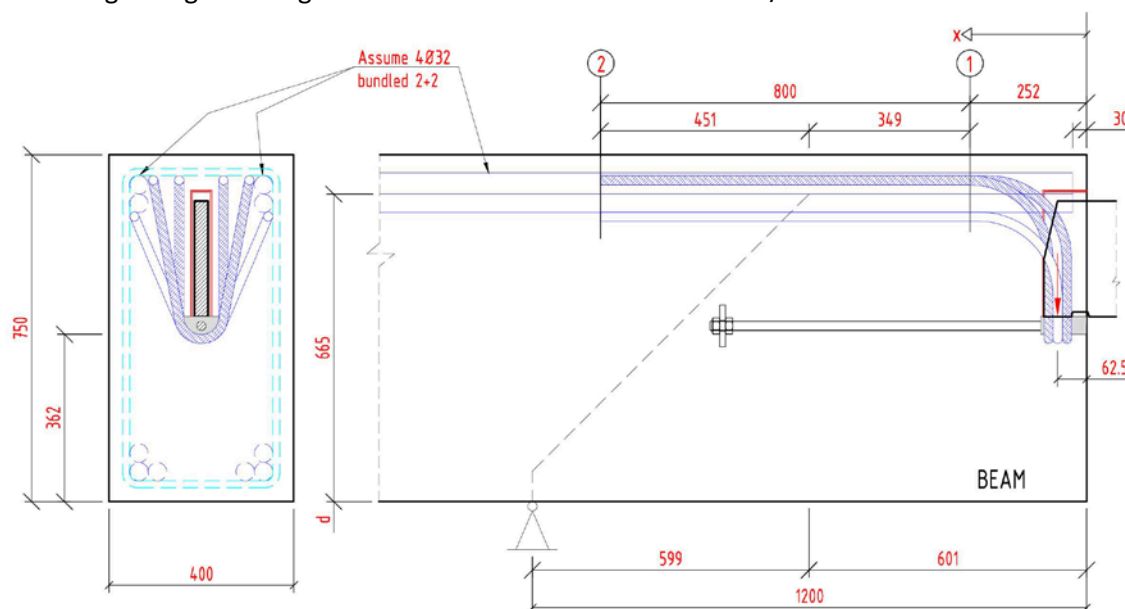
Horizontal anchoring of half round steel:  $R_H = 0,3 \times F_V = 135 \text{ kN}$ :

Select: 1×M20 threaded bars, 8.8 with nut & steel plate = 141kN

### 5.3 EXAMPLE – REINFORCEMENT IN BEAM END

Assume:

- Columns with six meters spacing. Beam-beam connection at 1,2m cantilevering from column.
- Cross section as illustrated in Figure 17.
- $z=0,9 \times d=0,9 \times 665\text{mm}=599\text{mm}$
- Horizontal part of the suspension reinforcement is 800mm ( $\approx$  equals the minimum calculated lap length). I.e. the bars end at  $x=252+800=1052\text{mm}$ . (The final required length is found from the calculations)
- Neglecting self-weight. Assumed dead and live loads = 0kN/m



**Figure 17: Example – Beam with BSF450 beam box. (Note, the illustrated reinforcement does not represent the conclusion from the evaluations, follow the calculations below.)**

### 5.3.1 REINFORCEMENT IN TOP OF BEAM- BOND AND ANCHORAGE

The tensile force in the reinforcement at top of the beam at distance  $z$  from the support:

$$S = \frac{450kN \times (1200 - 62,5)mm}{599mm} = 855kN$$

Estimate, required reinforcement:

$$A_s = 855kN / 435MPa = 1966mm^2$$

⇒ Assume main reinforcement at top of beam: 4Ø32 bundled 2+2 (=3216mm<sup>2</sup>)



Equivalent diameter of 2Ø32 bundled:

$$\varnothing_n = \varnothing \times \sqrt{2} = 32 \times \sqrt{2} = 45\text{mm}$$

Anchoring length of a bundle:

$$L_n = \frac{\pi \times 16^2 \times 435\text{MPa} \times 2}{\pi \times \varnothing_n \times f_{bd}} = \frac{\pi \times 16^2 \times 435\text{MPa} \times 2}{\pi \times 45 \times 2,79\text{MPa}} = \frac{700\text{kN}}{0,3944\text{kN/mm}} = 1774\text{mm}$$

Control 1: Anchoring at support: (x=1200mm):

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (1200 - 30)\text{mm} / 1774\text{mm} = 2121\text{mm}^2$$

$$A_{eqv} > 1966\text{mm}^2 \Rightarrow \text{OK.}$$

Control 2: Anchoring at distance z from support (x=1200-599=601mm).

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (601 - 30)\text{mm} / 1774\text{mm} = 1035\text{mm}^2$$

$$A_{eqv} < 1966\text{mm}^2 \Rightarrow \text{NOT OK}$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in Ø32:

$$S_{\varnothing 32} = A_{eqv} \times 435\text{MPa} = 1035\text{mm}^2 \times 435\text{MPa} = 450\text{kN}$$

$$\text{Not anchored: } \Delta S = 855\text{kN} - 450\text{kN} = 405\text{kN}$$

Required anchoring length 6Ø16:

$$L_n = \frac{405000\text{N}}{\pi \times \varnothing \times f_{bd} \times 6} = \frac{405000\text{N}}{\pi \times 16 \times 2,79\text{MPa} \times 6} = 481\text{mm}$$

Transfer of force to the main reinforcement with lap of bars. Select  $l_0 = 1,5 \times l_n = 1,5 \times 481\text{mm} = 721\text{mm}$

Available length:  $L_{\varnothing 16} = 451\text{mm}$ , see Figure 17.

$\Rightarrow$  Solution: Horizontal part of suspension reinforcement is elongated 300mm.

Control 3: Anchoring at the end of the suspension reinforcement (x=1052).

In the example, this point is within a distance z from the support. Thus, the tension and the required reinforcement will be as calculated in control 1:

Equivalent fully anchored reinforcement:

$$A_{eqv} = 3216\text{mm}^2 \times (1052 - 30)\text{mm} / 1774\text{mm} = 1852\text{mm}^2$$

$$A_{eqv} < 1966\text{mm}^2 \Rightarrow \text{NOT OK.}$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in Ø32:

$$S_{\varnothing 32} = A_{eqv} \times 435\text{MPa} = 1852\text{mm}^2 \times 435\text{MPa} = 805\text{kN}$$

Not anchored:  $\Delta S = 855 \text{ kN} - 805 \text{ kN} = 50 \text{ kN}$

Required anchoring length 6 $\phi$ 16:

$$L_n = \frac{50000 \text{ N}}{\pi \times \phi \times f_{bd} \times 6} = \frac{50000 \text{ N}}{\pi \times 16 \times 2,79 \text{ MPa} \times 6} = 60 \text{ mm}$$

$\Rightarrow$  This is ok if the horizontal part of the suspension reinforcement is elongated 300mm as stated in control 2.

Control 4: Bond/transfer of force into reinforcement at top of beam:

Increase in force per/mm:

$$\Delta S(x)/dx = (\Delta M(x)/dx)/z = V(x)/z = 450 \text{ kN}/599 \text{ mm} = 752 \text{ N/mm}$$

Capacity for increase in force by bond per/mm:

$$\Delta S_{\text{bond}}(x)/dx = f_{bd} \times \phi_n \times \pi \times 2 = 2,79 \times 45 \times \pi \times 2 = 788 \text{ N/mm}$$

$\Delta S_{\text{bond}}(x)/dx > \Delta S(x)/dx \Rightarrow \text{OK}$ . The bond to the main reinforcement is sufficient to take the increase in force.

### 5.3.2 SHEAR STIRRUPS IN BEAM END

Use a strut-and-tie model with compression diagonal at 45°. The shear at the end of the beam is  $F_v = 450 \text{ kN}$ . Beam as illustrated in Figure 17.

$$\frac{A_s}{s} = \frac{V_{Rd,s}}{z \times f_{yd}} \approx \frac{450 \times 10^3 \text{ N}}{0,599 \text{ m} \times 435 \text{ MPa}} = 1727 \text{ mm}^2 / \text{m}$$

Assume stirrup diameter  $\phi 12$ .

$\Rightarrow$  Select  $\phi 12 \text{ c}/100$  (2261 mm<sup>2</sup>/m)

### 5.3.3 SHEAR COMPRESSION IN BEAM END

Shear compression: EC2, clause 6.2.3. Beam as illustrated in Figure 17.

$$V_{Rd,max} = \alpha_{cw} \times b_w \times z \times u_1 \times f_{cd} / (\cot \theta + \tan \theta)$$

$$b_w = b_{\text{beam}} = 400 \text{ mm}$$

$$V_{Rd,max} = \{1,0 \times 400 \times 0,599 \times 0,6 \times [1 - (35/250)] \times 19,8 / (1+1)\} \times 10^{-3}$$

$$V_{Rd,max} = 1223 \text{ kN} (> V_{Rd} \Rightarrow \text{OK})$$

## 5.3.4 HORIZONTAL BARS IN BEAM END

Example: Beam as illustrated in Figure 17:

Narrow stirrups for horizontal force:

Assume  $z=0,9 \times d$

$$\frac{A_s}{s} \times h = \frac{F_v}{z \times f_{yd}} \times h = \frac{450000N}{0,9 \times 665mm \times 435MPa} \times 362mm = 626mm^2$$

Select three narrow u-bars:  $\emptyset 12 = \pi \times 6^2 \times 6 = 678mm^2$ . Placed just below the unit.

Simplified: Horizontal length of bar:  $L = (z - H) + 40\emptyset = (665 - 362)mm + 40 \times 12mm \approx 800mm$

Wide stirrups for splitting force:

$$A_s = \frac{1}{4} \times \frac{F_v}{f_{yd}} = \frac{1}{4} \times \frac{450000N}{435MPa} = 259mm^2$$

Select three u-bars:  $\emptyset 12 = \pi \times 6^2 \times 3 = 339mm^2$ . Distributed below the unit.

Simplified: Horizontal length of bar:  $L = 40\emptyset = 40 \times 12mm \approx 500mm$ .

## PART 6 - BSF 700

### 6.1 BEAM BOX – ANCHORING REINFORCEMENT

(Note: In the example calculations, «good» bond conditions are assumed when calculating  $f_{bd}$ . This may not be the case at the top of the beam, see EC2, clause 8.4.2 (2))

**1) Required cross section for reinforcement:**

$$A_s = \frac{F_v}{f_{yd}} = \frac{700kN}{435MPa} = 1609mm^2$$

$$2\emptyset 25 \text{ stirrups} = 490mm^2 \times 4 = 1960mm^2$$

$$\text{Capacity of selected reinforcement: } 1960mm^2 \times 435MPa = 852kN$$

**2) Mandrel diameter – Bending of reinforcement - EC2, clause 6.5.4:**

$$\emptyset_{mf, \min} = \frac{F_v}{b_{eff} \times 0,6 \times (1 - \frac{f_{ck}}{250}) \times f_{cd} \times 0,5} = \frac{700000}{550 \times 0,6 \times (1 - \frac{35}{250}) \times 19,8MPa \times 0,5} = 249mm$$

$b_{eff}$  = effective width of beam. Assume:  $b_{eff} = b_{beam} = 550mm$

$\emptyset_{mf}$  = Mandrel diameter of reinforcement.  
Concrete strut assumed in 45degrees, se Part 2.

⇒ Select:  $\emptyset=320\text{mm}$

### 3) Anchoring of reinforcement, EC2 clause 8.4.3 and 8.4.4:

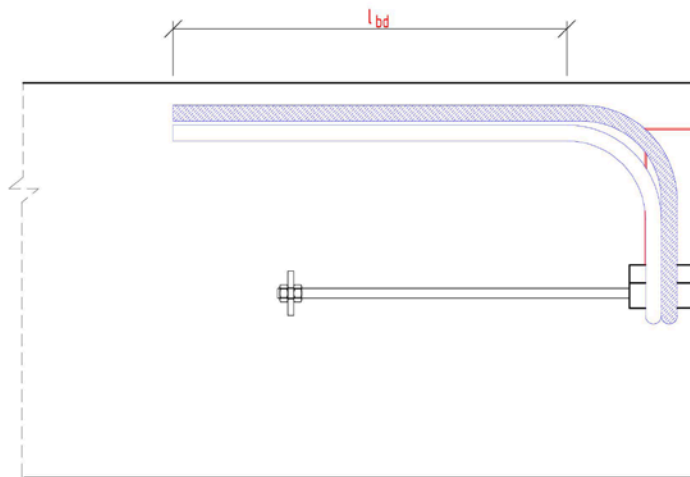


Figure 18: Anchoring of reinforcement.

$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\emptyset}{4} \times \frac{\sigma_{sd}}{f_{bd}}$$

$$\text{Stress in reinforcement: } \sigma_{sd} = \frac{700\text{kN}}{1960\text{mm}^2} = 357\text{MPa}$$

$$l_{b,reqd} = \frac{25}{4} \times \frac{357}{2,79} = 800\text{mm}$$

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \emptyset; 100\text{mm}) = 250\text{mm}$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \emptyset) / \emptyset$$

Neglecting any positive effect of concrete cover, selecting  $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting  $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$

$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,0 \times 800 \text{ mm} = 800 \text{ mm}$$

#### 4) Lap of stirrups, EC2 clause 8.7.3:

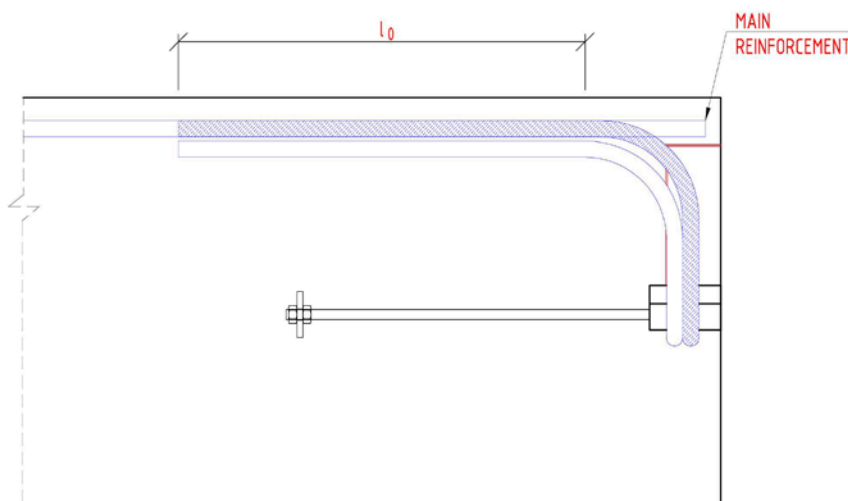


Figure 19: Lap of reinforcement.

$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, \text{reqd}} \geq l_{0, \text{min}}$$

Required lap length:

$$l_{b, \text{reqd}} = 800 \text{ mm, see evaluation in clause 3}$$

$$l_{0, \text{min}} = \max(0,3 \times \alpha_6 \times l_{b, \text{reqd}}; 15 \times \phi; 200 \text{ mm})$$

Table 8.2:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_5 = 1,0$  as calculated in clause 3.

Table 8.3:  $\alpha_6 = 1.5$  (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 800 \text{ mm} = 1200 \text{ mm}$$

$$\Rightarrow \text{Select: } l_0 = 1200 \text{ mm}$$

## 6.2 BEAM BOX – HORIZONTAL ANCHORING

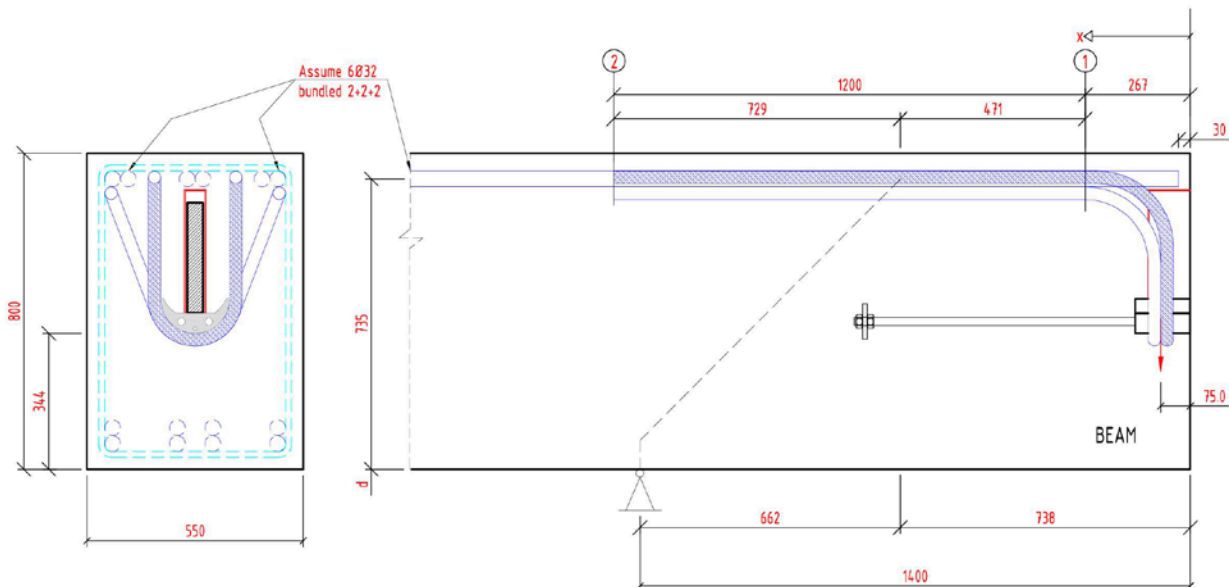
Horizontal anchoring of half round steel:  $R_H = 0,3 \times F_V = 210 \text{ kN}$ :

Select: 2×M20 threaded bars, 8.8 with nut & steel plate = 282 kN

### 6.3 EXAMPLE – REINFORCEMENT IN BEAM END

Assume:

- Columns with 7,2m spacing. Beam-beam connection at 1,4m cantilevering from column.
- Cross section as illustrated in Figure 20.
- $z=0,9 \times d=0,9 \times 735\text{mm}=662\text{mm}$
- Horizontal part of the suspension reinforcement is 1200mm ( $\approx$  equals the minimum calculated lap length). I.e. the bars end at  $x=267+1200=1467\text{mm}$ . (The final required length is found from the calculations)
- Neglecting self-weight. Assumed dead and live loads = 0kN/m.



**Figure 20: Example – Beam with BSF700 beam box. (Note, the illustrated reinforcement does not represent the conclusion from the evaluations, follow the calculations below.)**

#### 6.3.1 REINFORCEMENT IN TOP OF BEAM– BOND AND ANCHORAGE

The tensile force in the reinforcement at top of the beam at distance  $z$  from the support:

$$S = \frac{700\text{kN} \times (1400 - 75)\text{mm}}{662\text{mm}} = 1401\text{kN}$$

Estimate, required reinforcement:

$$A_s = 1401\text{kN} / 435\text{MPa} = 3220\text{mm}^2$$

$\Rightarrow$  Assume main reinforcement at top of beam: 6Ø32 bundled 2+2+2 (=4825mm<sup>2</sup>)

Equivalent diameter of 2Ø32 bundled:

$$\phi_n = \phi \times \sqrt{2} = 32 \times \sqrt{2} = 45mm$$

Anchoring length of a bundle:

$$L_n = \frac{\pi \times 16^2 \times 435MPa \times 2}{\pi \times \phi_n \times f_{bd}} = \frac{\pi \times 16^2 \times 435MPa \times 2}{\pi \times 45 \times 2,79MPa} = \frac{700kN}{0,3944kN/mm} = 1774mm$$

Control 1: Anchoring at support (x=1400mm):

Equivalent fully anchored reinforcement:

$$A_{eqv} = 4825mm^2 \times (1400-30)mm / 1774mm = 3726mm^2$$

$$A_{eqv} > 3220mm^2 \Rightarrow OK.$$

Control 2: Anchoring at distance z from support (x=1400-662=738mm).

Equivalent fully anchored reinforcement:

$$A_{eqv} = 4825mm^2 \times (738-30)mm / 1774mm = 1925mm^2$$

$$A_{eqv} < 3220mm^2 \Rightarrow NOT OK.$$

Selected solution in this case: Use of the suspension reinforcement. These bars will have sufficient anchoring towards the end of the beam, but anchoring towards the support must be evaluated:

Force anchored in  $\phi 32$ :

$$S_{\phi 32} = A_{eqv} \times 435MPa = 1925mm^2 \times 435MPa = 837kN$$

Not anchored:  $\Delta S = 1401kN - 837kN = 564kN$

Required anchoring length 4 $\phi 25$ :

$$L_n = \frac{534000N}{\pi \times \phi \times f_{bd} \times 4} = \frac{534000N}{\pi \times 25 \times 2,79MPa \times 4} = 609mm$$

Transfer of force to the main reinforcement with lap of bars. Select  $l_0 = 1,5 \times l_n = 1,5 \times 609mm = 914mm$

Available length:  $L_{\phi 25} = 729mm$ , see Figure 14.

$\Rightarrow$  Solution: Horizontal part of suspension reinforcement is elongated 200mm.

Control 3: Anchoring at the end of the suspension reinforcement (x=1467).

In the example, this point is on the inside of the support

$\Rightarrow$  The anchoring at x=1467mm is sufficient, see control 1. (The anchoring is further improved when the reinforcement is elongated 200mm as stated in control 2.

Control 4: Bond/transfer of force into reinforcement at top of beam:

Increase in force per/mm:

$$\Delta S(x)/dx = (\Delta M(x)/dx)/z = V(x)/z = 700kN/662mm = 1058N/mm$$

Capacity for increase in force by bond per/mm:

$$\Delta S_{bond}(x)/dx = f_{bd} \times \phi_n \times \pi \times 3 = 2,79 \times 45 \times \pi \times 3 = 1183N/mm$$

$\Delta S_{bond}(x)/dx > \Delta S(x)/dx \Rightarrow OK$ . The bond to the main reinforcement is sufficient to take the increase in force.

### 6.3.2 SHEAR STIRRUPS IN BEAM END

Use a strut-and-tie model with compression diagonal at 45°. The shear at the end of the beam is  $F_v=700\text{kN}$ . Beam as illustrated in Figure 20.

$$\frac{A_s}{s} = \frac{V_{Rd,s}}{z \times f_{yd}} \approx \frac{700 \times 10^3 \text{ N}}{0,662 \text{ m} \times 435 \text{ MPa}} = 2430 \text{ mm}^2 / \text{m}$$

Assume stirrup diameter  $\emptyset 12$ .

⇒ Select  $\emptyset 12\text{c}/\text{c}80$  ( $2827 \text{ mm}^2/\text{m}$ ).

### 6.3.3 SHEAR COMPRESSION IN BEAM END

Shear compression: EC2, clause 6.2.3. Beam as illustrated in Figure 20.

$$V_{Rd,max} = \alpha_{cw} \times b_w \times z \times u_1 \times f_{cd} / (\cot \theta + \tan \theta)$$

$$b_w = b_{\text{beam}} = 550 \text{ mm}$$

$$V_{Rd,max} = \{1,0 \times 550 \times 662 \times 0,6 \times [1 - (35/250)] \times 19,8 / (1+1)\} \times 10^{-3}$$

$$V_{Rd,max} = 1860 \text{ kN} (> V_{Rd} \Rightarrow \text{OK})$$

### 6.3.4 HORIZONTAL BARS IN BEAM END

Example: Beam as illustrated in Figure 20.

Narrow stirrups for horizontal force:

Assume  $z=0,9 \times d$

$$\frac{A_s}{s} \times h = \frac{F_v}{z \times f_{yd}} \times h = \frac{700000 \text{ N}}{0,9 \times 735 \text{ mm} \times 435 \text{ MPa}} \times 245 \text{ mm} = 596 \text{ mm}^2$$

Select three narrow u-bars:  $\emptyset 12 = \pi \times 6^2 \times 6 = 678 \text{ mm}^2$ . Placed just below the unit.

Simplified: Horizontal length of bar:  $L = (z - H) + 40\emptyset = (665 - 362) \text{ mm} + 40 \times 12 \text{ mm} \approx 800 \text{ mm}$

Wide stirrups for splitting force:

$$A_s = \frac{1}{4} \times \frac{F_v}{f_{yd}} = \frac{1}{4} \times \frac{700000 \text{ N}}{435 \text{ MPa}} = 402 \text{ mm}^2$$

Select two u-bars:  $\emptyset 16 = \pi \times 8^2 \times 2 = 402 \text{ mm}^2$ . Distributed below the unit.

Simplified: Horizontal length of bar:  $L = 40\emptyset = 40 \times 16 \text{ mm} \approx 700 \text{ mm}$



REVISION	
Date:	Description:
21.10.2013	First edition
30.06.2014	Changed the half round steel on the BSF700 unit. Corrected Table 3.
20.08.2014	Changed position of the M20 threaded bars in the half round steel BSF 700 unit. Changed steel plate anchoring M20 threaded bars BSF 700 unit.
13.01.2015	Updated Table 4. Required thread length in blind holes.
27.02.2015	Included a nut on the front side of the steel plate anchoring the threaded bars. (To ensure correct position of the plate when casting the concrete).
24.05.2016	New template